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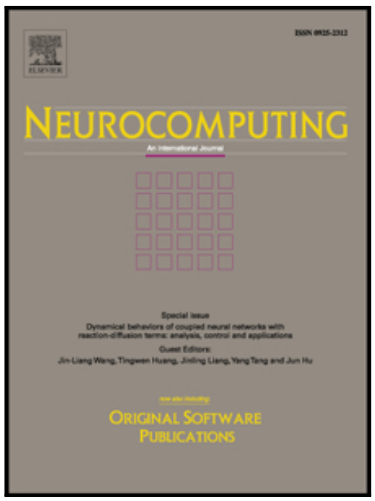
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A Combined Robust Fuzzy Time Series Method for Prediction of Time Series

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Highlights

- Outlier(s) have an adverse impact on the performance of fuzzy time series models
- We proposed a combined robust fuzzy time series model (C-R-FTSM)
- C-R-FTSM uses fuzzy inputs composed of membership values as well as the crisp data
- Training process of C-R-FTSM is performed by PSO in a single optimization process
- Huber's loss function based on M estimator is used as robust fitness function

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A Combined Robust Fuzzy Time Series Method for Prediction of Time Series

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Abstract

In case of outlier(s) it is inevitable that the performance of the fuzzy time series prediction methods is influenced adversely. Therefore, current prediction methods will not be able to provide satisfactory accuracy rates for defuzzified outputs (predictions) when the data has outlier(s). In this study, not only to be able to sort out this problem but also to be able to improve the forecasting accuracy, we propose a combined robust approach for fuzzy time series by assessing how the prediction performance of the methods will be affected from the outlier(s). In the proposed model, different from the current models, both crisp values and membership values are used as inputs and also real time series observations are taken as outputs. The proposed model therefore does not require defuzzification transaction and uses single multiplicative neuron model to determine the fuzzy relations and a robust fitness function in its training process. While performing the training process of this model by particle swarm optimization within a combined single optimization process, using crisps values and membership values together provides successful results by getting further information. The various implementations are illustrated to show that the proposed model could obtain more accurate and robust results in forecasting.

Keywords: fuzzy time series; robust model; forecasting; multiplicative neuron model; Huber's loss function

1 Introduction

It is become clear that getting accurate forecasting results has an incontrovertible effect in our daily life. The classical time series methods which have been used for forecasting cannot handle forecasting problems in which not only the data are imprecise and vague but also the values of time series are linguistic terms represented by fuzzy sets. Moreover, requiring some strict assumptions such as stability, invertibility, sufficient number of observations may lead

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to researchers to use fuzzy time series (FTS) forecasting methods which have an extensive area of utilization such as information technologies, economy, finance, ecological science and hydrology. Song and Chissom [81] were the pioneers of studying such problems and have proposed FTS model in 1993 which is capable of dealing with incomplete and vague data under uncertain circumstances by applying the theory of fuzzy logic. From this point of view, some kind of fuzzy time series forecasting methods have been commonly used for time series prediction during the last few decades or so.

Fundamentally, the methods of FTS are based on three stages as fuzzification, identification of fuzzy relations and defuzzification. Each of these stages has an effective role on forecasting performance of the methods.

Although FTS methods have a huge literature, there are still some problems needed to be figured out. It should be pointed out that in particular when the data has outlier(s) or extreme value(s), model structures will get affected locally close to the outlier(s). In this case, while discarding outlier(s) could be systematically problematic and may cause information loss, analysing data with outlier(s) can lead to wrong prediction and modeling as well. The real life data sets with outlier(s) have led researchers to use robust techniques. In the literature the first concept of robustness was used by Box with its statistical meaning [16]. And the history of robust regression estimators which based on $\mathbf{L}_1$ estimation method was proposed by Boscovish [15]. While with the improvement of computer language, Tukey [86], Huber [62] and Andrews [9] started to study on different robust techniques, in particular, M-type estimators suggested by Huber [62] have provided a basis for robust regression analysis.

However in recent years, there has been a renewed research interest on some robust techniques and to the best knowledge of the authors, there is no any study seen in the literature which evaluates FTS with outlier(s). In this respect, suggestion of a robust FTS prediction model can be considered as a fundamental request in FTS literature.

Apart from robustness problem, there is another issue needed to be solved relevant to fuzzy time series steps. FTS approaches proposed in the literature consider its own three steps that constitute the solution process as separate processes. This situation may cause a rise in the modeling error due to the sum of the error that may occur in each step. In this regard, synchronous assessment of the steps constituting the analysis process will produce a single modeling error and will lead to a reduction in the modeling error.

In this study, we proposed a combined robust fuzzy time series model (C-R-FTSM) for prediction of time series. By using this combined robust prediction model, error that may occur in the analysis process of data is minimized. In the combined model, the formulas of

FCM technique are used for the fuzzification whereas single multiplicative neuron model uses to determine the fuzzy relations and a robust fitness function in its training process. In the proposed C-R-FTSM, while the model inputs are composed of the real observations of time series and membership values, the real observations constitute the target. It is worth mentioning that though much work has been done to reduce the modeling error, in this study thereby using more information obtained from data we have managed to get minimum modeling error even in case of outlier(s). Moreover, determination of relevant parameters, in other words the training process of this model is performed by particle swarm optimization (PSO) within a single optimization process. In this optimization process Huber's loss function based on M estimator is used as fitness function. On top of the robustness characteristic of the proposed prediction model, it can be mentioned three distinguished features of the combined model when it is compared with the current FTS models; the first one is that the proposed prediction model uses fuzzy inputs composed of the membership values in conjunction with the crisp data as in fuzzy inference systems, the second one is that since the combined model generates the crisp data as outputs, the defuzzification transaction is not necessary and also the third outstanding speciality of the model is that whole prediction process is performed within a single process because the centre of fuzzy sets which is utilized to get fuzzy inputs (memberships) and the weights and biases of the single multiplicative neuron model used in identification of relations between inputs and outputs are reached in a single optimization process. To evaluate the performance of the proposed C-R-FTSM, various time series are analyzed in case of both with and without outlier and obtained results are interpreted together.

Considering all of these, it can be mentioned three basic distinguished contributions of this paper; this paper is the first study that investigates the prediction performance of the fuzzy time series models in case of outlier(s) and offers a robust fuzzy time series prediction model, this paper is also the first work that takes the three stages of fuzzy time series analysis process into consideration as a single optimization process. It therefore ensures better prediction results in both presence and absence of outlier(s), Moreover, this study introduces a fuzzy time series prediction model using all membership values and crisp data information as an input and so it gets lower modeling error under favor of this further information.

The rest of this paper is organized as follows: In Section 2, a literature review is given in detail. Section 3 represents some basic definitions and notions of FTS, PSO and SMNM-ANN. The proposed model is introduced with an algorithm in Section 4. The experimental results are summarized and discussed in Section 5. Finally, the last section includes our

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conclusion and future work.

2 Literature Review

There are lots of studies which evaluate each of FTS analysis stages in a different way by using some well-known data. Enrolment data is one of these data types that has quite a wide range of usage areas in literature from the beginning of studies [4, 12, 19, 23, 24, 27, 29, 30, 32-34, 37, 38, 44, 51, 53, 54, 55-58, 63, 65, 67, 72, 75, 78, 80, 82-84, 87, 89, 95, 97]. In addition Taiwan Futures Exchange (TAIFEX) [4, 12, 32, 58, 59, 65, 68, 91] Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) [17-19, 25-28, 33, 36, 38-40, 42, 57, 59-61, 73, 93- 95, 99-101] and Index 100 in stock and bonds exchange market of Istanbul (IEX) [4] have been also widely utilized for getting accurate prediction in FTS literature. And also, for temperature prediction, various FTS forecasting methods were used in the same area [25, 31, 32, 42, 68-70, 73, 80, 91].

The partition of universe of discourse is one of the most common used methods in the first stage of methods called fuzzification. As is known the length of intervals affects forecasting accuracy in FTS methods. Consequently, determining optimal interval length in these methods is the main problem in some studies. However, interval lengths are determined constantly, Song and Chissom [81-83] and Chen [23, 24] determined equal interval lengths arbitrarily whereas Huarng [57] used average and distribution based and Egrioglu et al. [50, 52] used optimization based methods. Moreover, Huarng and Yu [59] improved the method by utilizing the ratio-based model from the viewpoint of opinion that inconstant interval length can improve the forecasting performance. Optimization of ratio has been put forward by Yolcu et al. [97]. Recently, some heuristic optimization algorithms have been used to get better results by Kuo et al. [65, 66], Davari, et al. [43], Park et al. [77] and Hsu et al. [56], Chen and Chung [29], Lee et al. [69,70]. Chen [20] and Chen and Chen [21] preferred to use entropy-based partitioning approach to get intervals in his high-order forecasting model. Wang et al. [90] took the temporal information into account to partition the universe of discourse. And also Cai et al. [18] preferred to use ant colony optimization to obtain a suitable partition of universe of discourse. Lu et al. [76] carried out a model for interval prediction based on granules information. In some studies fuzzy C-means (FCM) technique was used by Cheng et al. [38], Li et al. [74], Aladag et al. [6], Alpaslan et al. [8], Egrioglu [45], Egrioglu et al. [53] and Sun et al. [85]. While Askari et al. [11] and Askari and Montazerin [10] introduced forecasting models used fuzzy clustering algorithms, Wang and

Liu [92] presented an approach based on automatic clustering. Cheng et al. [41] utilized the K-means clustering algorithm to cluster subscripts of the fuzzy sets.

The stage of determination of fuzzy relation is a significant stage that the inner-relation of fuzzy time series is determined. This stage has a crucial effect on stating an appropriate model and obtaining superior forecasting performance. All of these affect lead researchers to use different methods in this case. Song and Chissom [81-83] used fuzzy logic relation matrix to determine the fuzzy relations. Instead of using fuzzy logic relation matrix, Sullivan and Woodall [84] used transition matrices consisted of Markov chain. Chen [23] proposed a new model by simplifying complex matrix calculations based on fuzzy group relation tables. On the other hand artificial neural networks (ANNs) have become another important fuzzy relation assignation tool. Uslu et al. [87] took recurrence number of the fuzzy relations into account in the stage of defuzzification.

Huarng and Yu [31] proposed a first order FTS approach using feed forward artificial neural network (FFANN) in this stage. Besides, Aladag et al. [2] developed the method of Huarng and Yu [31] and proposed a high order FTS forecasting model. During the determination of fuzzy relations, the membership values have been ignored and index numbers of fuzzy sets have been considered in all of these approaches. Several hybrid models to forecast different kind of time series were proposed by Wei et al. [94], Wang and Xiong [88], Egrioglu [46], Lee and Hong [71] and Chen and Chen [22, 28]. Huarng and Yu [60], Aladag et al. [2, 3], Egrioglu et al. [48-49] used FFANN in the identification of fuzzy relations. The ANN model which does not require to determine the number of units in the hidden layer was proposed by Aladag et al. [1] named as single multiplicative neuron model (SMNM-ANN). Also Yu and Huarng [100, 101] used forecasting models which consider all membership values and determine these values subjectively in order to cope with the loss of information and negative effects of this situation on the performance. Alpaslan and Cagcag [7], Alpaslan et al. [8], and Yolcu et al. [98] used FCM technique instead of determining the membership values subjectively. The use of ANN in identification of fuzzy relations has many advantages and disadvantages as well. Determination of unit number in hidden layer (architecture structure) and excessive number of parameters to be used during the analysis are the most prominent ones. Aladag [1] eliminated this problem by using artificial neural network with SMNM-ANN in the determination of fuzzy relations, but in this study fuzzy relations were determined subjectively and membership values were not considered. Bas et al. [13] introduced a fuzzy time series model with network structure.

In the last process of analysing FTS called defuzzification stage, many researchers as Chen

[23], Huarng [57] and Huarng and Yu [59] used centroid method. Song and Chissom [83] used ANN in this stage and Cheng et al. [37] and Aladag et al. [3] preferred to use adaptive expectation method. Yolcu et al. [98] proposed an approach considering all membership values in this stage.

3 Related Methodologies

3.1 Fuzzy time series

The fuzzy time series were firstly introduced by Song and Chissom [81]. The fuzzy time series definitions are given below by Song and Chissom [81].

Definition 1 Let $Y(t)$ ($t = 0, 1, 2, \ldots$), a subset of real numbers, be the universe of discourse on which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t)$, $f_2(t)$, … then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2 Suppose $F(t)$ is caused by $F(t-1)$ only, i.e., $F(t-1) \rightarrow F(t)$. Then this relation can be expressed as $F(t) = F(t-1) \circ R(t, t-1)$ where $R(t, t-1)$ is the fuzzy relationship between $F(t-1)$ and $F(t)$, and $F(t) = F(t-1) \circ R(t, t-1)$ is called the first order model of $F(t)$. " ° " represents max-min composition of fuzzy sets.

Definition 3 Suppose $R(t, t-1)$ is a first order model of $F(t)$. If for any t, $R(t, t-1)$ is independent of t , i.e., for any t , $R(t, t-1) = R(t-1, t-2)$, then $F(t)$ is called a time invariant fuzzy time series otherwise it is called a time variant fuzzy time series.

Song and Chissom [81] firstly introduced an algorithm based on the first order model for forecasting time invariant $F(t)$. In Song and Chissom [81] the fuzzy relationship matrix $R(t, t-1) = R$ is obtained by many matrix operations. The fuzzy forecasts are obtained based on max-min composition as below:

$$F(t) = F(t-1) \circ R \tag{1}$$

The dimension of R matrix is dependent number of fuzzy sets which are the partition number of universe and discourse. If we want using more fuzzy sets, we need different matrix operations to obtain the R matrix.

3.2 Particle swarm optimization

PSO is an evolutionary computation technique proposed by Kennedy and Eberhart [64]. PSO can be evaluated as a population based optimization tool. The particle swarm concept originated as a simulation of simplified social system [79]. Distinguishing feature of this

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heuristic algorithm is that it simultaneously examines different points in different regions of the solution space to obtain the global optimum solution. Local optimum traps can be avoided because of this feature of the method [5].

3.3 Single multiplicative neuron model

In neurons of feed-forward neural networks, the input signal is calculated based on addition function. Yadav et al. [96] proposed a single multiplicative neuron model. In the model, the input signal of the neuron is estimated by the multiplication function. Yadav et al. [96] showed that single multiplicative neuron model gives better forecasting performance for time series forecasting. Zhao and Yang [102] recommended the use of PSO instead of back propagation learning algorithm proposed by Yadav et al. [96] in the training of single multiplicative neuron model. The structure of single multiplicative neuron model has a single neuron and unlike feed-forward neural network, multiplication is performed to the signal coming into the neuron.

3.4 Fuzzy C-means clustering

Song and Chissom [81] used decomposition of universal discourse in the stage of fuzzification. However, there are several problems related to the decomposition of universal discourse in the stage of fuzzification such as the decision on what the number of intervals will be, arbitrar determination of interval length and the arbitrary choice of membership degrees. In order to overcome these problems, in some studies, fuzzy C-means (FCM) clustering method is used for fuzzification. FCM clustering method is first introduced by Bezdek [14]. This is a most widely used clustering algorithm. In the proposed model, the formulas of FCM technique (see in Section 4 / Step 4) are used for the fuzzification i.e. to get the fuzzy inputs of the model which are composed of the membership values.

4 The Proposed Model

In the fuzzy time series literature, although the models have outstanding prediction performance, there is no study that investigates the prediction performance of the model in case of outlier(s). The usage of a robust technique has become crucial not only in order to get better forecasting results but also to be able to evaluate the real data sets with outlier(s) properly.

Moreover another point needed to be solved in FTS approaches is reducing the error that

may occur in each step of analysis process. In this direction we proposed a combined robust fuzzy time series prediction model. In our proposed method we aim;

- To obtain robust fuzzy time series prediction model by considering existing outlier(s) values' adverse effect on the basic stages of FTS method.
- To get better prediction results by carrying out the three steps of analysis process in a single optimization process, synchronously.
- To design much better model not only using all membership values and crisp data information as an input but also minimizing modeling error.

And it is also expected that this approach will make a significant contribution as a first study that has these features in its area.

In the proposed method, there is no need to have defuzzfication step since the crisp observations of data are used as targets of the model. And also the transactions of fuzzification and identification of fuzzy relation are performed through PSO in a single optimization process. In this process, the optimal centre of fuzzy sets that provide to get membership values by using the formulas of FCM and weights of SMNM-ANN that produce the optimal outputs are obtained. In this optimization process, Huber's loss function based on M-estimator which is a robust measure of distance between the targets and the outputs of the model is used as a fitness function. It must be mentioned that this optimization process can be counted as a training process of the combined robust model.

The main advantages of the proposed method can be summarized as follows:

- The C-R-FTSM is a first robust FTSM in the literature. Moreover, The C-R-FTSM has a characteristic that it is virtually not affected by outlier (s).
- The C-R-FTSM does not need subjective judgments for fuzzification as it uses equations based on fuzzy clustering method.
- The C-R-FTSM takes the advantage of the flexible modeling ability of ANN as it uses SMNM-ANN in identification of fuzzy relation and so it does not have the problem of determining the number of units in hidden layer on the contrary MLP has.
- The C-R-FTSM can be considered as a Fuzzy Inference System owing to the use of the lagged crisp data as well as the membership values as inputs of the model. This feature of the model enables to get less modeling error by assessment of further information.
- The C-R-FTSM does not need defuzzification transaction as it uses real observations as targets of the model. In the C-R-FTSM, the modeling errors which will be able to be separately formed in every step are demoted a single model error by carrying out in a

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single optimization process.

The algorithm of the C-R-FTSM can be given step-by-step as follows.

Algorithm: ***The C-R-FTSM***

Step 1: Define the parameters of the model

We can mention two basic parameter groups in analysis process as the number of inputs and the parameters of PSO.

The number of lagged crisp data and the membership values of the first lagged observations are determined as inputs. Moreover, the parameters of PSO algorithm $(c_1, c_2, w, pn, maxiter, vm_1, vm_2, vm_3)$ are specified in this step. Where c_1 is cognitive coefficient, c_2 is social coefficient, w is inertia parameter, pn the number of particle, $maxiter$ is the maximum number of iterations, vm_1 is the velocity of each particle for centres of fuzzy sets (v_i), vm_2 is are the velocity of weights (w_i) and biases (b_i) of SMNM-ANN and vm_3 is the velocity of, the scale parameter.

Step 2: Generate the initial position and velocities of each of particle with dimensions of $(3 \times c) + (2 \times q) + 1$

For each particle id, initial positions and velocities are randomly generated and kept in vectors X_{id}, and V_{id} given as follows:

$$X_{id} = \{x_{id,1}, x_{id,2}, \cdots, x_{id,(3\times c)+(2\times q)+1}\} \quad (2)$$

$$V_{id} = \{v_{id,1}, v_{id,2}, \cdots, v_{id,(3\times c)+(2\times q)+1}\} \quad (3)$$

where $x_{id,1}, x_{id,2}, \cdots, x_{id,c}$ represent the centres of fuzzy sets and also $x_{id,c+1}, x_{id,c+2}, \cdots, x_{id,2\times c}$ and $x_{id,2c+1}, x_{id,2c+2}, \cdots, x_{id,3\times c}$ represent the weights and biases of SMNM-ANN, respectively. Moreover, $x_{id,3c+1}, x_{id,3c+2}, \cdots, x_{id,(3\times c)+q}$ and $x_{id,(3\times c)+q+1}, x_{id,(3\times c)+q+2}, \cdots, x_{id,(3\times c)+(2\times q)}$ represent the weights and biases of SMNM-ANN corresponding to the crisp inputs, respectively. And also, $x_{id,(3\times c)+(2\times q)+1}$ represents the scale parameter. The initial positions of each particle in a swarm are randomly generated from the uniform distribution $\big(min(X(t)), max(X(t))\big)$, (0,1), and (6,10) for centres of fuzzy sets, weights and biases of SMNM-ANN and the scale parameter, respectively. The velocities are also randomly generated from uniform distribution $(-vm_1, vm_1)$, $(-vm_2, vm_2)$ and $(-vm_3, vm_3)$. Where $min(X(t))$ and $max(X(t))$ are the minimum and maximum values of organized time series, respectively.

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Step 3: Set the number of iterations $iter$ to one $(iter = 1)$

Step 4: Obtain the membership values $\left(\mu_{L_{it}}, i = 1,2,\cdots,c\right)$

To get the membership values for each observation, by using the first c positions of each particle, the equation given in (4) and based on FCM method is utilized.

$$\mu_{L_{it}} = \frac{1}{\sum_{k=1}^{c}\left(\frac{d(X(t),v_i)}{d(X(t),v_k)}\right)^{2/(b-1)}}, i = 1,2,\cdots,c\ ;\ \ t = 1,2,\cdots,T \tag{4}$$

Where b $(b > 1)$ is a scalar termed the weighting exponent and controls the fuzziness of the resulting clusters. $\mathrm{d}(X(t), v_i)$ is a similarity measure (Euclidian distance) between an observation and the centre of corresponding fuzzy cluster. The fuzzy clustering of objects is described by a fuzzy matrix μ with T rows and c columns in which T is the number of data objects and c is the number of clusters. μ_{it}, the element in the i^{th} row and t^{th} column in μ, indicates the degree of association or membership function of the i^{th} object with the t^{th} cluster. The characters of μ are as follows:

$$\mu_{L_{it}} \in [0,1] \quad \forall\, i = 1,2,\cdots,c\ ;\ \forall\, t = 1,2,\cdots,T \tag{5}$$

$$\sum_{i=1}^{c}\mu_{L_{it}} = 1 \quad \forall\, t = 1,2,\cdots,T \tag{6}$$

$$0 < \sum_{t=1}^{T}\mu_{L_{it}} < T \quad \forall\, i = 1,2,\cdots,c \tag{7}$$

Step 5: Obtain the output values of SMNM-ANN

The output values of SMNM-ANN are obtained for training data by using the number of $(2 \times c) + (2 \times q)$ positions of each particle constituted from weights $\left(w_i^{(1)} \text{ and } w_j^{(2)}\right)$ and biases $\left(b_i^{(1)} \text{ and } b_j^{(2)}\right)$ of SMNM-ANN.

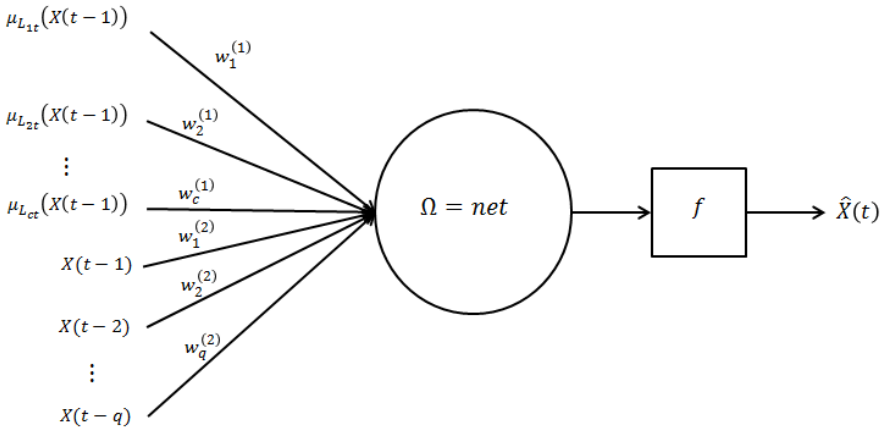


Fig. 1. The structure of SMNM-ANN

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In Fig. 1, $\mu_{L_{it}}(t-1)$ and $X(t-j)$ are the membership value representing the degree of observation belonging to c^{th} fuzzy sets at $(t-1)$ and the lagged crisp observations which constitute the inputs of network, respectively. Ω function comprises multiplication of the weighted inputs and is obtained by equation (8). In addition, f denotes the activation function whereas $\hat{X}(t)$ represents outputs of the model. Output of the model is calculated as in equation (9).

$$\Omega(\mu, w, b) = net = \left\{\prod_{i=1}^{c}\left[w_i^{(1)} \times \mu_{L_i}\left(X(t-1)\right) + b_i^{(1)}\right]\right\} \times \left\{\prod_{j=1}^{q}\left[w_j^{(2)} \times X(t-\mathrm{j}) + b_j\right]\right\} \quad (8)$$

$$\hat{X}(t) = f(net) = \frac{1}{1+exp(-net)} \quad (9)$$

In the case that the number of fuzzy sets defined for the defuzzification is c, there are $(3 \times c) + (2 \times q) + 1$ variables to be optimized by PSO. The structure of a particle is shown as in Fig. 2.

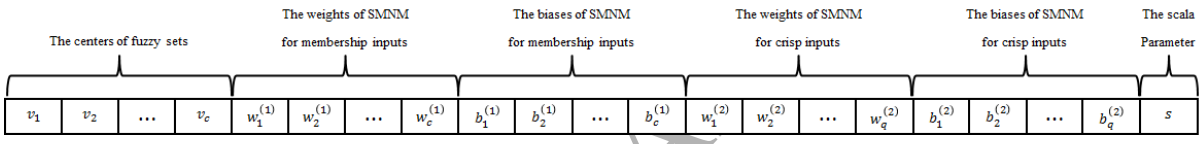


Fig. 2. The structure of a particle

where, $v_i, b_i, i = 1,2,\cdots,c$ are centres of fuzzy clusters and $w_i^{(1)}, w_j^{(2)}, b_i^{(1)}$ and $b_j^{(2)}$, $i = 1,2,\cdots,c$; $b_i, j = 1,2,\cdots,q$ represent weights and biases of SMNM-ANN.

Step 6: Calculate fitness function values

Huber's loss function of each particle id is calculated as the objective value $(Fitness_{id})$. Calculation of $Fitness_{id}$ for each particle id can be given as follow:

$$(Fitness_{id}) = \sum_{t=1}^{T} \tilde{e}_{t,id} \quad (10)$$

where T represents the number of learning sample for SMNM-ANN and $\tilde{e_t}$ symbolises Huber's error value and it is calculated as in below:

$$\tilde{e}_t = \begin{cases} {e_{t,id}^2}/{2} & , \quad if \quad |e_{t,id}| \le \beta_{id} \\ \beta_{id}\left|e_{t,id} - {\beta_{id}}^2/2\right| & , \quad otherwise \end{cases}$$

Let the difference between target and output of the model be $e_t = X_{id}(t) - \hat{X}_{id}(t)$, $t = 1,2,..,T$. β_{id} can be obtained as follow:

$$\beta_{id} = c_{id} * \underset{t,id}{\text{median}}\left(e_{t,id} - \underset{t,id}{\text{median}}(e_{t,id})\right)$$

Where $X_{id}(t)$ and $\hat{X}_{id}(t)$ are target and forecasting value of the historical data for particle id, respectively. c_{id} represents the scale parameter.

Step 7: Update the personal best position vector P_{id} $(3 \times c) + (2 \times q) + 1$

The personal best position vector $P_{id} = [p_{id,1}, p_{id,2}, \cdots, p_{id,(3\times c)+(2\times q)+1}]$ of each particle id is updated with its position vector $X_{id} = [x_{id,1}, x_{id,2}, \cdots, x_{id,(3\times c)+(2\times q)+1}]$ if its objective value at the current iteration $iter$ is smaller than its objective value at the previous iteration $(iter - 1)$ by letting $p_{id,1} = x_{id,1}$, $p_{id,2} = x_{id,2}$, ⋯, and $p_{id,(3\times c)+(2\times q)+1} = x_{id,(3\times c)+(2\times q)+1}$.

Step 8: Choose the best particle P_{gbest} among all particles value

Step 9: Update the velocity and the position vectors of each particle

The updating of the velocity vector V_{id} and the position vector X_{id} of each particle id are carried out as follows:

$$v_{id,k}^{iter+1} = \left[w \times v_{id,k}^{k} + c_1 \times rand_1 \times \left(p_{id,k} - x_{id,k}\right) + c_2 \times rand_2 \times \left(P_{gbest,k} - x_{id,k}\right)\right] \quad (11)$$

$$x_{id,k}^{iter+1} = x_{id,k}^{t} + v_{id,k}^{t+1}\ , k = 1,2,\cdots,(3 \times c) + (2 \times q) + 1 \quad (12)$$

where, $rand_1$ and $rand_2$ are randomly generated from uniform distribution with zero and one parameters.

Step 10: Check the stopping criterion

If the current $iter < maxiter$, then do $iter = iter + 1$ and go to Step 4 else go to next step.

Step 11: Determine the optimal values of variables

Let the position vector $P_{gbest} = [p_{gbest,1}, p_{gbest,2}, \cdots, p_{gbest,(3\times c)+(2\times q)+1}]$ of the best particle $gbest$ be the optimal centre of fuzzy sets, weights of SMNM-ANN and scale parameter.

The flowchart of the proposed C-R-FTSM with single optimization process is given in Fig. 3.

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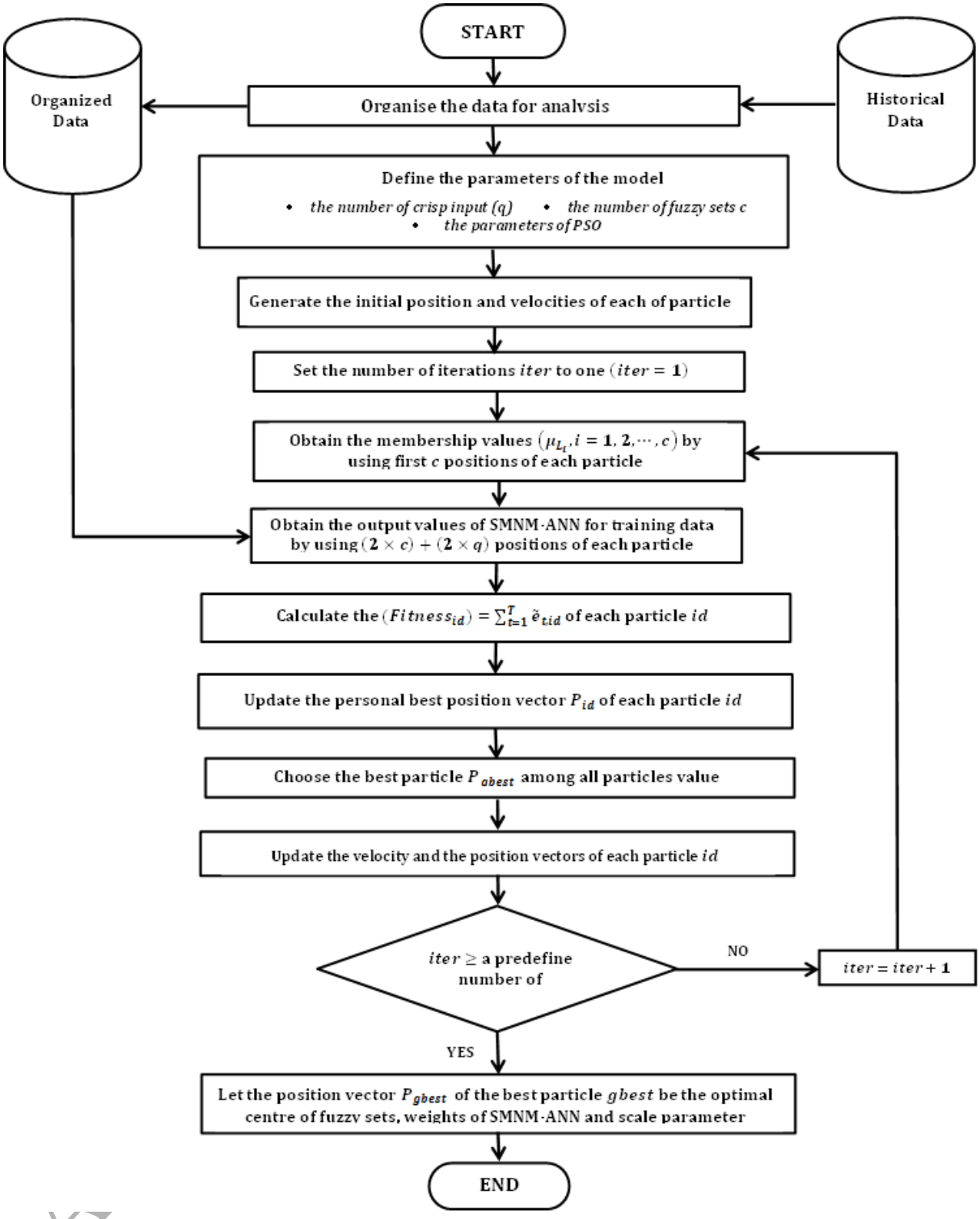


Fig. 3. Flowchart of the proposed C-R-FTSM

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5 Experimental Results and Discussion

5.1 Data preparation

We consider to analyze the daily Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) in 2000, 2001, 2002, 2003 and 2004 TAIEX and Istanbul Stock Index (IEX) in 2009, 2010, 2011 and 2012 data sets to evaluate the combined robust forecasting model. In many studies proposed in forecasting problems, a convenient ratio to separate in-samples from out-of-samples ranging from 70%:30% to 90%:10%. We chose two different parts of data for training. The periods of the first nine and ten months of data are used for training of the model (the in-sample) and the periods of the last two and three months of data are utilized as validation sets (the out-of-sample) to assess the model performance. The size of data sets, training sets and the validation sets can be seen in Table 1.

Table 1
The summarization of data organization.

Data	The Size of Data	Validation Set 1 (Last Two Months)			Validation Set 2 (Last Two Months)		
		The Size of		The Ratio (%)	The Size of		The Ratio (%)
		Training Set	Validation set		Training Set	Validation set	
TAIEX-2000	271	224	47	83/17	201	70	74/26
TAIEX-2001	244	201	43	82/18	180	64	74/26
TAIEX-2002	248	205	43	83/17	183	65	74/26
TAIEX-2003	249	206	43	83/17	184	65	74/26
TAIEX-2004	250	205	45	82/18	185	65	74/26
IEX-2009	250	209	41	84/16	189	61	76/24
IEX-2010	247	207	40	84/16	188	59	76/24
IEX-2012	251	210	41	84/16	190	61	76/24
IEX-2012	251	208	43	83/17	189	62	75/25

The data sets are further analyzed for two cases to investigate the models' performance with outliers as well as without outliers. For this purpose, 3 and 5 percent of the relevant data sets are contaminated with outliers. The outliers are generated by taking 10 times of randomly determined observations and the data modified by replacing the original observations with generated outliers. For the graph of TAIEX-2000 data both original case and 3 percent contaminated case with outliers can be seen from Figs. 4 and 5.

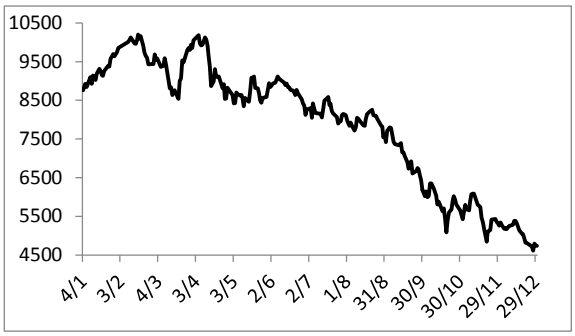


Fig. 4. TAIEX-2000 data

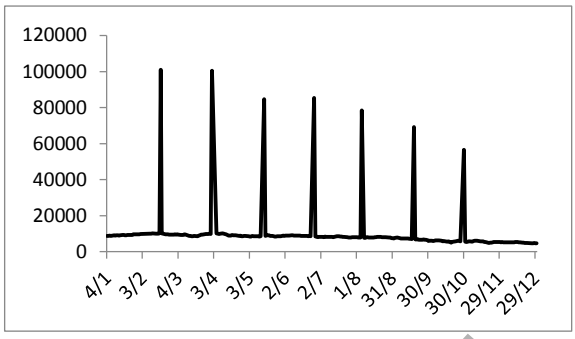


Fig. 5. %3 contaminated TAIEX-2000 data

5.2 Defining the relevant parameters of the model

- *Define the inputs of the forecasting model*

The inputs of the forecasting model are composed of the lagged crisp data and the membership values of the first lagged observations. The number of fuzzy set, in other words the number of membership inputs, is determined form the range of 3 to 10 and we take the crisp inputs by ranging the number of lagged crisp data from 2 to 20.

- *Define the parameters of PSO*

The parameters of PSO are determined as $c_1 = 2$, $c_2 = 2$, $w = 0.9$, $pn = 30$, $maxiter = 200$, $vm_1 = 100$, $vm_2 = 1$ and $vm_3 = 0.1$.

5.3 Performance measure

Throughout the literature there are many different measures that have been used to evaluate the performance of models on FTS. So suitable comparisons can be realized between the proposed robust model and those reported in the literature by adopting the most commonly used metric; root mean square error (RMSE).

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(X(t) - \hat{X}(t)\right)^2} \qquad (10)$$

where T represents the number of forecasts, $X(t)$ and $\hat{X}(t)$ are the real observation and forecasting value of the historical data at time *t*, respectively.

To compare the performance of the C-R-FTSM in case of outliers, we use some current models in the literature: SC93 [82], C96 [23], H01[1] [57] (average based), H01[2] [57] (distribution based), HY06 [59], A09 [2], Y13 [98], and E15 [13].

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5.4 TAIEX data analysis

Firstly daily TAIEX data belonging to five years are analyzed through combined robust forecasting models and obtained results are evaluated with various models' results, together.

Case 1: *Original TAIEX Data Analysis*

In this phase, the proposed C-R-FTSM is applied to five original TAIEX data sets. And also the best forecasting results of the C-R-FTSM for the original TAIEX data sets are presented with the results of some well-known models in Tables 2 and 3 for the validation set 1 and validation set 2, respectively.

Table 2
Performance evaluations of the methods for original TAIEX data - validation set 1.

	RMSE					Average
Methods	2000	2001	2002	2003	2004	RMSE's
[82]	293.30	115.97	75.70	76.59	81.65	128.64
[23]	224.80	115.97	75.70	76.59	81.65	114.94
[57][1]	472.55	359.08	234.09	246.91	384.41	339.41
[57][2]	472.55	809.56	115.56	307.93	384.41	418.00
[59]	132.74	124.41	82.25	62.06	84.69	97.23
[58]	152.00	130.00	84.00	56.00	116.00	107.60
[61]	154.42	124.02	93.48	65.51	72.35	101.96
[100]	131.00	130.00	80.00	58.00	67.00	93.20
[2]	167.79	119.77	75.7	57.52	63.39	95.71
[25]	129.42	113.33	66.82	53.51	60.48	84.71
[26]	123.62	115.33	71.01	58.06	57.73	85.15
[35]	119.98	114.47	67.17	52.49	52.27	81.28
[36]	131.25	112.55	65.77	52.23	54.17	83.19
[50]	255.00	130.00	84.00	56.00	116.00	128.20
[98]	226.63	102.00	66.19	50.76	55.36	100.19
[28]	124.52	114.66	64.71	52.84	52.96	81.94
[18]	131.53	112.59	60.33	51.54	**50.33**	81.26
[41]	125.62	113.04	62.94	51.46	54.25	81.46
[13]	140.45	119.75	77.35	59.81	59.16	91.30
C-R-FTSM	**115.36**	**105.90**	**59.98**	**47.22**	50.60	**75.81**

Considering Tables 2 and 3, with respect to the RMSE criterion, it is clearly seen that forecasting performance of the proposed C-R-FTSM is better than other methods in the literature for both validation sets except TAIEX-2004 data in which the proposed model has the second best performance. Even for TAIEX-2004, the C-R-FTSM's results are just 0.54 and 0.74 percent worse than the best results and it means that the combined robust model, from this aspect, has a quite competitive characteristic with regard to the forecasting ability of it. Moreover, it must be noticed that the C-R-FTSM shows outstanding forecasting

performance in terms of average of RMSE criterion. Fig. 6 represents the forecasting performance of the models as on average RMSE values of whole data sets for validation set 1 and validation set 2.

Table 3
Performance evaluations of the methods for original TAIEX data - validation set 2.

Methods	RMSE					Average RMSE's
	2000	2001	2002	2003	2004	
[82]	435.17	130.67	118.84	130.91	77.22	178.56
[23]	169.59	130.67	92.95	79.21	77.22	109.93
[57][1]	291.01	472.31	391.10	337.78	403.35	379.11
[57][2]	446.11	1444.63	1302.81	824.60	403.35	884.30
[59]	153.74	115.73	102.05	63.55	75.44	102.10
[60]	152.00	130.00	84.00	56.00	72.35	98.87
[61]	154.42	124.02	95.73	70.76	55.91	100.17
[2]	193.92	115.53	92.94	78.88	60.86	108.43
[101]	149.59	98.91	78.71	58.78	**53.83**	87.96
[98]	307.58	119.67	98.28	74.29	60.73	132.11
[13]	169.65	110.02	85.34	63.68	63.59	98.46
C-R-FTSM	**141.87**	**98.89**	**75.72**	**55.74**	54.22	**85.29**

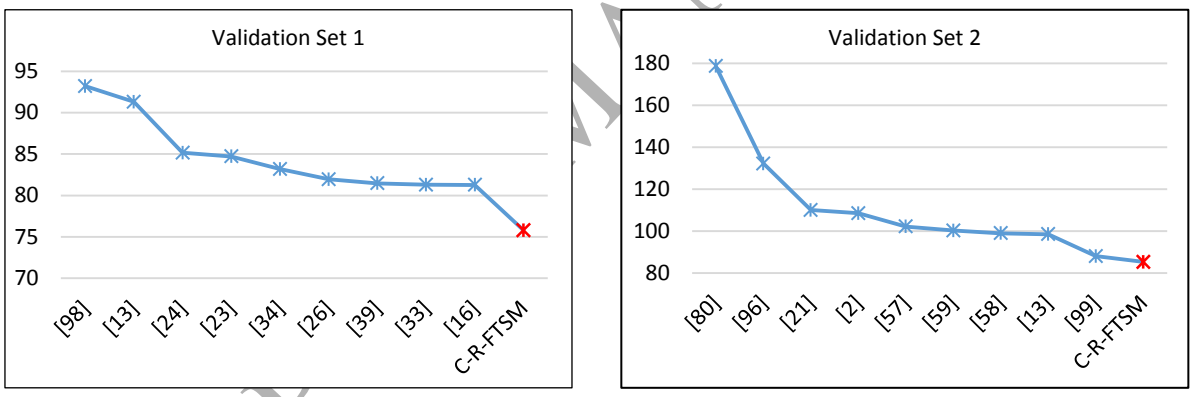


Fig. 6. The forecasting performance of the models as on average RMSE values

The line and the scatter graphs of forecasts obtained from the proposed C-R-FTSM and the real observations are given in Figs. 7-16.

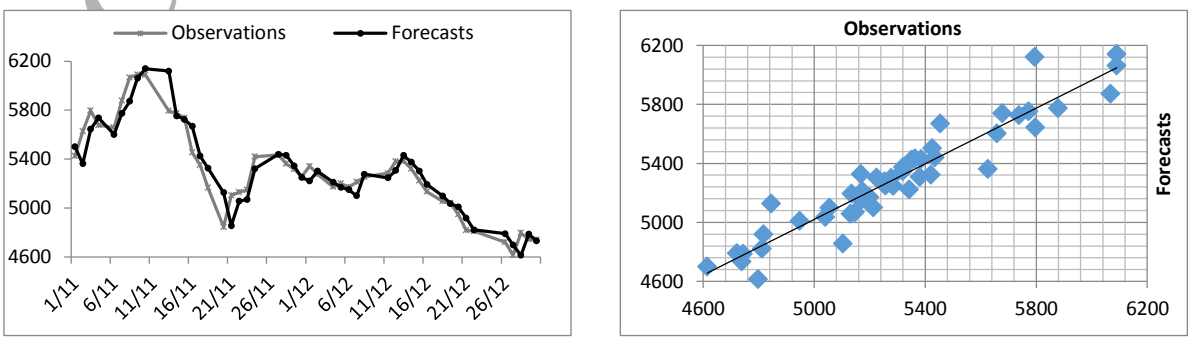


Fig. 7. The out-of-sample forecasts and observations for TAIEX-2000 validation set 1

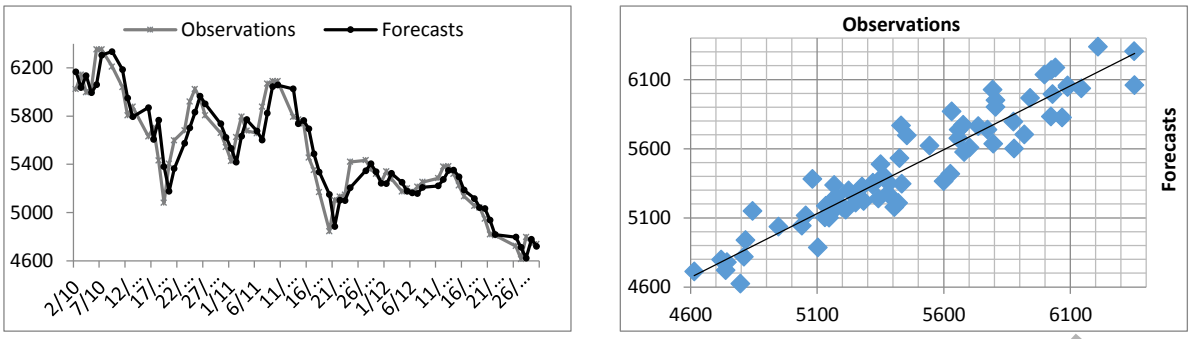


Fig. 8. The out-of-sample forecasts and observations for TAIEX-2000 validation set 2

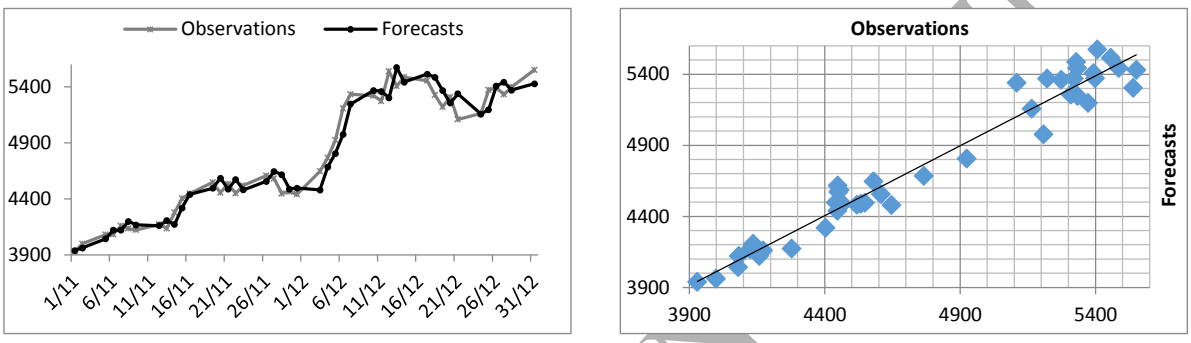


Fig. 9. The out-of-sample forecasts and observations for TAIEX-2001 validation set 1

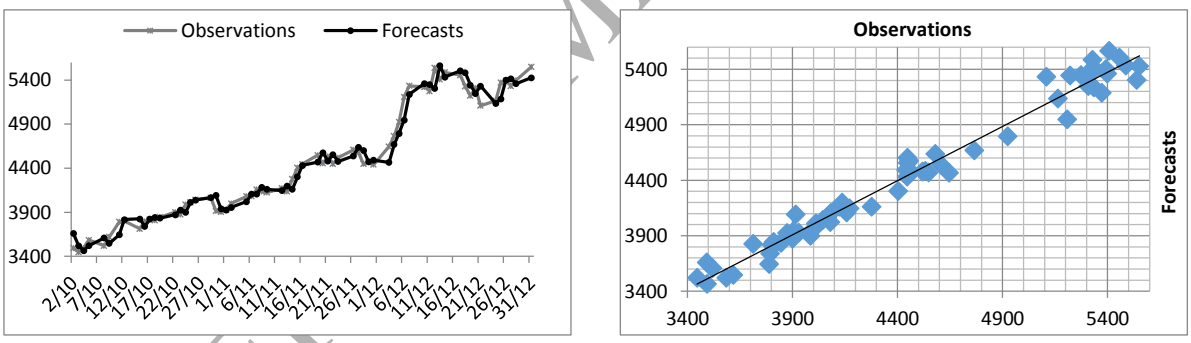


Fig. 10. The out-of-sample forecasts and observations for TAIEX-2001 validation set 2

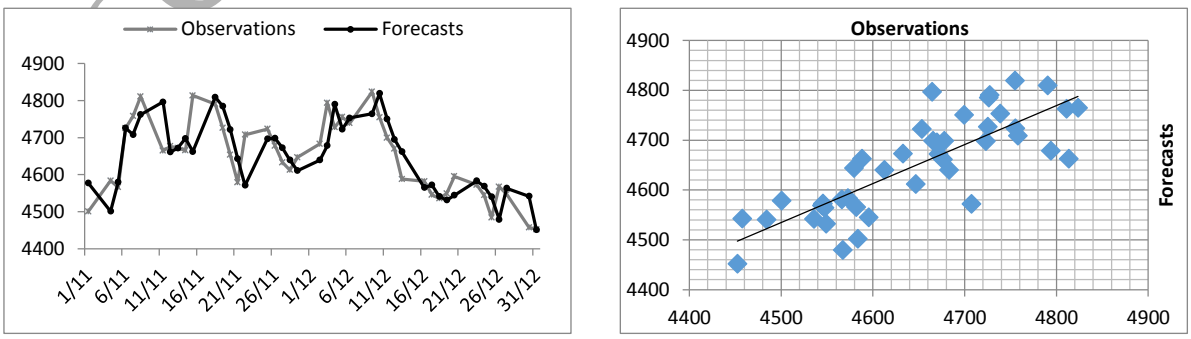


Fig. 11. The out-of-sample forecasts and observations for TAIEX-2002 validation set 1

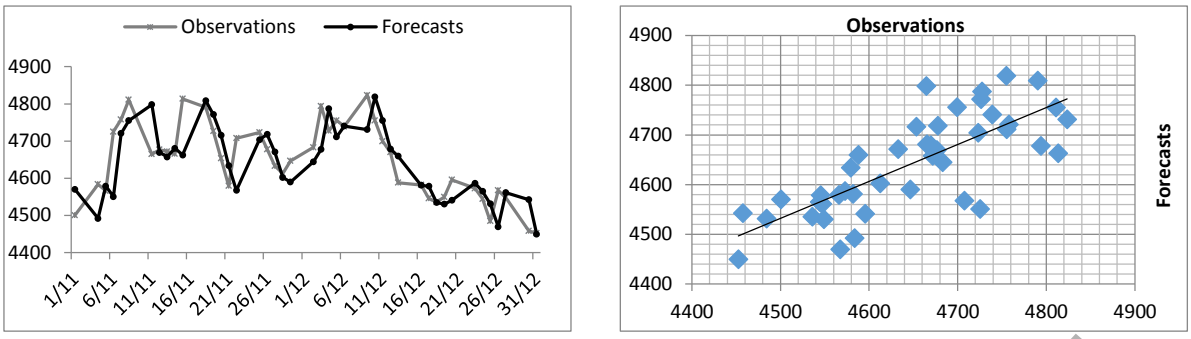


Fig. 12. The out-of-sample forecasts and observations for TAIEX-2002 validation set 2

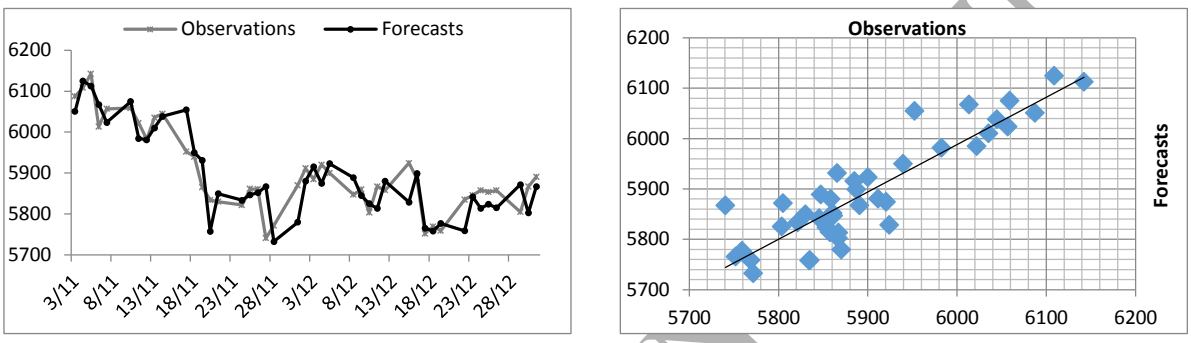


Fig. 13. The out-of-sample forecasts and observations for TAIEX-2003 validation set 1

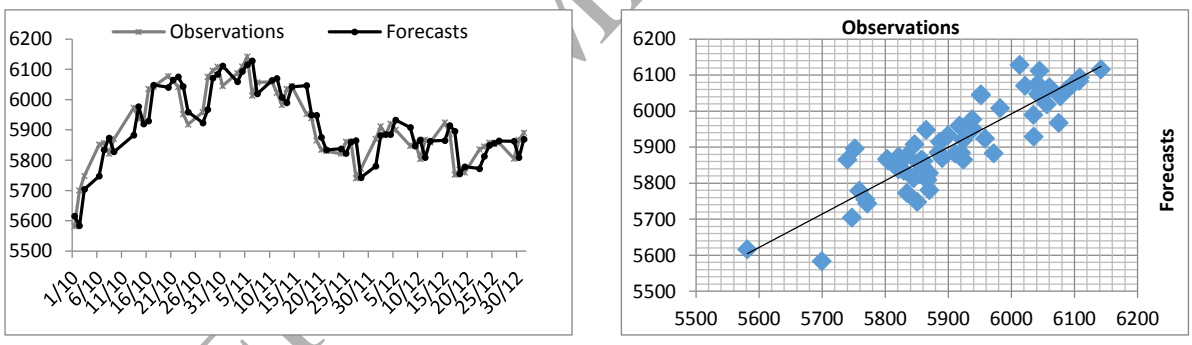


Fig. 14. The out-of-sample forecasts and observations for TAIEX-2003 validation set 2

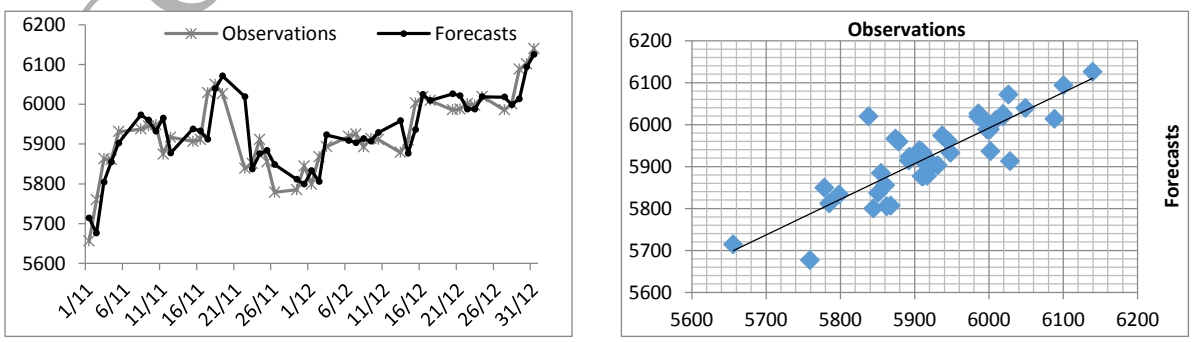


Fig. 15. The out-of-sample forecasts and observations for TAIEX-2004 validation set 1

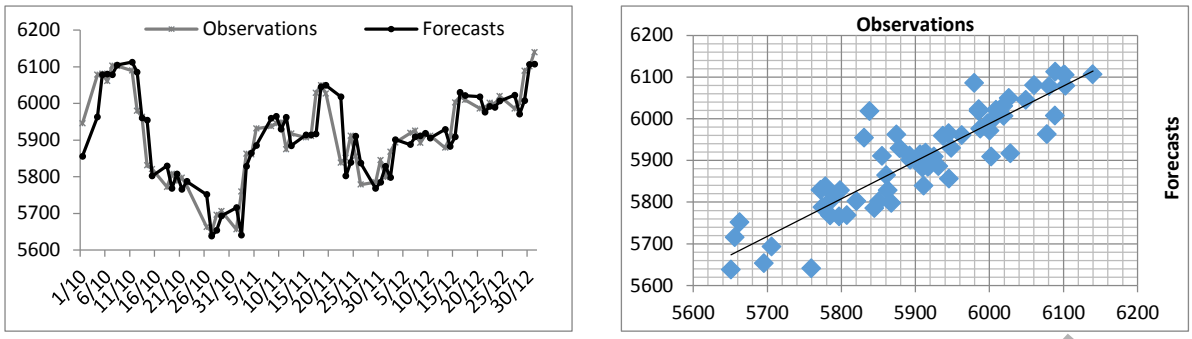


Fig. 16. The out-of-sample forecasts and observations for TAIEX-2004 validation set 2

All of the line graphs show that the forecasts of the proposed C-R-FTSM are quite compatible with the observations. Another evidence of the superior performance of the C-R-FTSM can be seen from the scatter plots which are the mathematical diagrams using Cartesian coordinates to display values of forecasts obtained from the proposed robust model and the real observations. These scatter plots demonstrate that the most of points are in proximity to line segment as it is expected.

Case 2: *3% contaminated TAIEX Data Analysis*

In this case, the TAIEX data contaminated with outliers at the rate of three percent are considered to observe the performance of the models in case of outliers. In consequence of the analyzes, for both validation sets, obtained forecasting errors and percentage variation of RMSE criterion values in comparison with the original data analysis are represented in Tables 4 and 5.

Table 4
Performance evaluations of the methods for 3% contaminated TAIEX data - validation set 1.

	TAIEX2000		TAIEX2001		TAIEX2002		TAIEX2003		TAIEX2004	
Models	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**
SC93 [82]	11931.53	3968.03%	10100.34	8609.44%	10820.49	14193.91%	12525.26	16253.65%	14178.7	17265.22%
C96 [23]	3686.25	1539.79%	5516.67	4656.98%	5205.32	6776.25%	9646.88	12495.48%	5831.89	7042.55%
H01 [57][1]	8050.72	1603.68%	16594.11	4521.28%	14397.35	6050.35%	29792.09	11965.97%	35206.11	9058.48%
H01 [57][2]	25299.59	5253.84%	26231.98	3140.28%	3148.56	3148.57%	33161.38	10669.13%	46039.42	11876.64%
HY06 [59]	5278.43	3876.52%	2821.47	2167.88%	4020.02	4787.56%	5474.07	8720.61%	3529.53	4067.59%
A09 [2]	368.26	119.48%	158.06	31.97%	78.56	3.78%	90.31	57.01%	63.39	9.71%
Y13 [98]	242.93	7.19%	114.45	12.21%	71.17	7.52%	63.04	24.19%	59.75	7.93%
E15 [13]	185.84	32.32%	144.72	20.85%	119.87	54.97%	87.22	45.83%	153.61	159.65%
C-R-FTSM	**122.45**	**6.15%***	**106.35**	**0.42%***	**62.11**	**3.55%***	**51.69**	**9.47%***	**51.53**	**1.84%***

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Table 5
Performance evaluations of the methods for 3% contaminated TAIEX data - validation set 2.

	TAIEX2000		TAIEX2001		TAIEX2002		TAIEX2003		TAIEX2004	
Models	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**
SC93 [82]	437.68	0.58%	12989	9840.31%	118.84	**0.00%***	34110.1	25956.15%	23578.36	30434.01%
C96 [23]	170.98	0.82%	5998.15	4490.30%	95.16	2.38%	3017.4	3709.37%	8489.73	10894.21%
H01 [57][1]	292.89	0.65%	20363.45	4211.46%	611.00	56.23%	25582.72	7473.78%	33885.75	8301.08%
H01 [57][2]	448.93	0.63%	27458.84	1800.75%	33613.64	2480.09%	36606.04	4339.25%	44719.03	10986.90%
HY06 [59]	158.52	3.11%	2124.19	1735.47%	110.70	8.48%	70.15	10.39%	3896.76	5065.38%
A09 [2]	268.54	38.48%	125.86	8.94%	102.51	10.30%	102.87	30.41%	67.39	10.73%
Y13 [98]	342.08	11.22%	139.46	16.54%	107.48	9.36%	82.79	11.44%	64.86	6.80%
E15 [13]	242.17	42.75%	121.25	10.21%	120.31	40.96%	77.35	21.47%	152.98	140.57%
C-R-FTSM	**141.93**	**0.04%***	**99.12**	**0.23%***	**77.34**	**2.14%**	**55.97**	**0.41%***	**57.31**	**5.70%***

An examination of Tables 4 and 5 clearly shows that while the performance of other models is adversely affected by the outliers, the proposed C-R-FTSM is virtually never influenced by outliers. From these tables, it is also clearly seen that the other models have rather big variation rates with even more than 100%. Conversely, the variance rates obtained from the proposed model, for all cases, are 3 and less than 3 percent apart from three cases with variance rates 5.70%, 6.15% and 9.47% which can be regarded as reasonable variation rates.

Case 3: *5% contaminated TAIEX Data Analysis*

In this case, the TAIEX data contaminated with outliers at the rate of five percent are also examined to reveal the effect of outliers on the models' performance. Forecasting errors and percentage variation of RMSE obtained in this case for validation sets 1 and 2 are given in Tables 6 and 7, respectively.

Table 6
Performance evaluations of the methods for 5% contaminated TAIEX data - validation set 1.

	TAIEX2000		TAIEX2001		TAIEX2002		TAIEX2003		TAIEX2004	
Models	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**
SC93 [82]	8634.83	2844.03%	20586.35	17651.44%	16413.05	21581.70%	3623.56	4631.11%	30156.86	36834.30%
C96 [23]	4827.65	2047.53%	8417.15	7158.04%	2151.28	2741.85%	84.64	10.51%	14146.18	17225.39%
H01 [57][1]	9177.92	1842.21%	25858.98	7101.45%	20819.29	8793.71%	31940.29	12836.01%	38497.77	9914.77%
H01 [57][2]	29299.23	6100.24%	30052.78	3612.24%	34402.29	-70.25%	33600.78	10811.82%	48122.75	12418.60%
HY06 [59]	169.88	27.98%	3464.21	2684.51%	4266.04	5086.67%	74.86	20.63%	1139.57	1245.58%
A09 [2]	310.81	85.24%	156.97	31.06%	82.61	9.13%	61.81	**7.46%***	63.39	9.71%
Y13 [98]	251.69	11.06%	114.42	12.18%	76.54	15.64%	56.35	11.01%	64.45	16.42%
E15 [13]	271.37	93.21%	165.32	38.05%	128.92	66.67%	94.08	57.30%	174.62	195.17%
C-R-FTSM	**126.99**	**10.08%***	**106.35**	**0.42%***	**62.59**	**4.35%***	**52.17**	10.48%	**52.54**	**3.83%***

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Table 7
Performance evaluations of the methods for 5% contaminated TAIEX data - validation set 2.

	TAIEX2000		TAIEX2001		TAIEX2002		TAIEX2003		TAIEX2004	
Models	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**	**RMSE**	**Variance Rate**
SC93 [82]	437.68	**0.58%***	17252.76	13103.31%	28365.58	23768.71%	36518.84	27796.14%	33736.29	43588.54%
C96 [23]	170.98	0.82%	8014.78	6033.60%	6655.04	7059.81%	8240.25	10303.04%	12019.06	15464.70%
H01 [57][1]	292.89	0.65%	24108.59	5004.40%	14402.48	3582.56%	36082.34	10582.20%	41441.26	10174.27%
H01 [57][2]	448.93	0.63%	31953.73	2111.90%	30613.72	2249.82%	37791.94	4483.06%	48885.68	12019.92%
HY06 [59]	155.55	1.18%	1436.87	1141.57%	5055.85	4854.29%	4368.29	6773.78%	5811.15	7603.01%
A09 [2]	316.48	63.20%	356.9	208.92%	105.44	13.45%	112.62	42.77%	71.95	18.22%
Y13 [98]	335.00	8.91%	158.78	32.68%	121.30	23.42%	80.43	8.26%	77.32	27.32%
E15 [13]	272.96	60.90%	142.69	29.69%	123.50	44.70%	93.1	46.20%	176.83	178.08%
C-R-FTSM	**149.6**	5.45%	**99.14**	**0.25%***	**77.16**	**1.90%***	**56.02**	**0.50%***	**56.85**	**4.85%***

Tables 6 and 7 indicate that the results obtained from the proposed C-R-FTSM in case of outliers generated by contaminating the original data at the rate of 5 percent are also very close to the results obtained from the original data sets. The variance rates of the proposed model's results, for all data set, are around 5% and less than 5% except just two cases with variance rates around 10%, which can be counted as reasonable variation rates, too. Moreover when we consider the RMSE values obtained for contaminated data sets, for all cases, we are informed that the proposed C-R-FTSM has still superior forecasting performance as well as its outstanding robustness feature. On the other hand, we can summarize the whole results averagely as in Table 8. From these outlined results, it is noticed that the proposed C-R-FTSM is almost never affected by outliers with 1.75% and 2.59% variation rate in comparison with the original data analysis for the validation set 2. Even for validation set 1, the proposed model shows a non-objectionable alteration with around 5% variation rate. The variation rates of the models which have the lowest four values are visually represented in Fig. 17.

Table 8
The average variation rates for whole TAIEX data.

	Contaminated Rate			
	3% - Case 1		5% - Case 2	
Models	**Validation set 1**	**Validation set 2**	**Validation set 1**	**Validation set 2**
SC93 [82]	12058.05%	13246.21%	16708.52%	21651.46%
C96 [23]	6502.21%	3819.42%	5836.66%	7772.39%
H01 [57][1]	6639.95%	4008.64%	8097.63%	5868.81%
H01 [57][2]	6222.43%	3921.53%	6574.53%	4173.07%
HY06 [59]	4724.03%	1364.56%	1813.07%	4074.77%
A09 [2]	44.39%	19.77%	28.52%	69.31%
Y13 [98]	11.81%	11.07%	13.26%	20.12%
E15 [13]	62.72%	51.19%	90.08%	71.91%
C-R-FTSM	**4.29%***	**1.71%***	**5.83%***	**2.59%***

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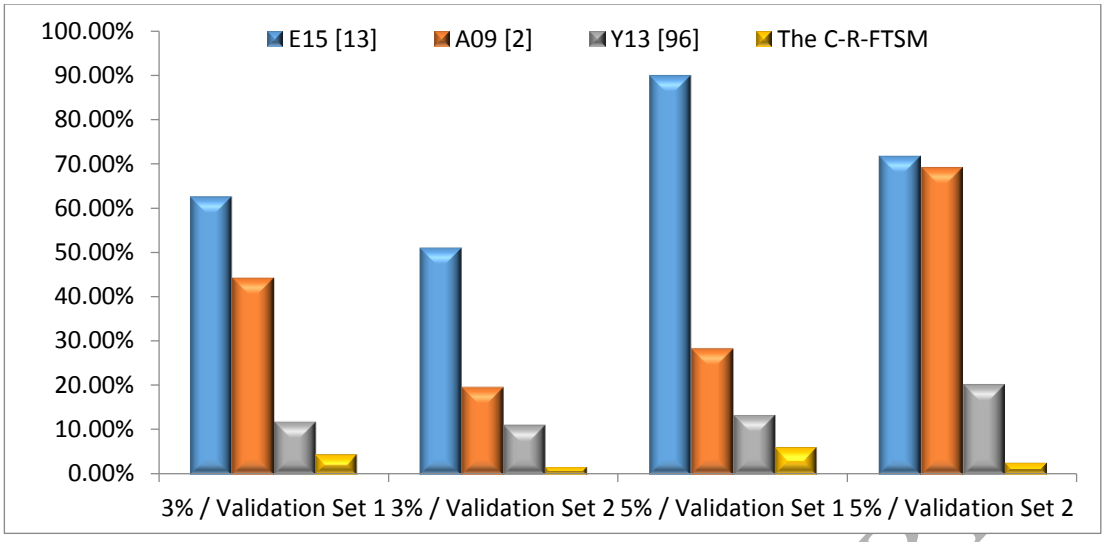


Fig. 17. The average variation rates of the models having the four lowest values – TAIEX data

5.5 IEX data analysis

Secondly four different daily IEX data composed of the years of 2009, 2010, 2011 and 2012 are examined via the C-R-FTSM and obtained results are compared with the results obtained from the other various models. In this process, two outlier cases which consist of *3% and 5% contaminated data sets are evaluated as well as the case of original data.* The best forecasting results obtained from analysis of original IEX data sets are summarized in Table 9. The robust model has best forecasting performance for overall IEX data sets. And also the best forecasting results of the models and percentage variation of RMSE in comparison with the original data analysis are presented in Tables 10-13 in case 3% and 5% contamination rate for the validation sets 1 and 2, respectively.

Table 9
Performance evaluations of the methods for original IEX.

Models	IEX2009		IEX2010		IEX2011		IEX2012	
	Validation set 1	Validation set 2	Validation set 1	Validation set 2	Validation set 1	Validation set 2	Validation set 1	Validation set 2
SC93 [82]	1028.60	2961.62	1064.53	3050.28	1105.33	1051.41	4289.12	6451.82
C96 [23]	1028.60	1510.67	1064.53	1013.33	1105.67	1051.41	930.87	1255.24
H01 [57][1]	1035.07	928.77	1027.74	889.53	1114.43	1118.63	669.32	631.88
H01 [57][2]	835.66	908.63	1348.90	917.47	985.07	1053.30	693.73	601.59
HY06 [59]	890.30	858.27	1115.40	915.78	981.15	994.54	654.27	609.63
A09 [2]	707.87	986.14	1096.77	927.72	929.77	977.45	815.56	963.23
Y13 [98]	811.10	1048.14	988.07	1136.61	898.62	960.05	921.46	1621.42
E15 [13]	832.75	911.29	1070.67	1023.95	968.33	1015.97	699.50	667.47
C-R-FTSM	**652.82***	**808.69***	**901.24***	**871.65***	**825.26***	**847.75***	**607.04***	**575.17***

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Table 10
Performance evaluations of the methods for 3% contaminated IEX data - validation set 1.

Models	IEX2009		IEX2010		IEX2011		IEX2012	
	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate
SC93 [82]	15647.61	1421.25%	22747.08	2036.82%	9842.97	790.50%	12654.67	195.04%
C96 [23]	97950.05	9422.66%	42578.25	3899.72%	65872.67	5857.72%	991.16	6.48%
H01 [57][1]	292415.39	28150.78%	188151.95	18207.35%	167344.18	14916.12%	2802.00	318.63%
H01 [57][2]	290748.77	34692.71%	314311.41	23201.31%	430671.22	43619.86%	5541.55	698.81%
HY06 [59]	44788.91	4930.77%	30216.65	2609.04%	25170.03	2465.36%	886.06	35.43%
A09 [2]	1527.09	115.73%	1260.00	14.88%	1289.56	38.70%	1088.43	33.46%
Y13 [98]	930.28	14.69%	1121.38	13.49%	1035.49	15.23%	1064.68	15.54%
E15 [13]	1267.56	52.21%	1332.77	24.48%	1331.18	37.475	751.74	7.47%
C-R-FTSM	**687.62**	**5.33%***	**948.12**	**5.20%***	**826.67**	**0.17%***	**619.49**	**2.05%***

Table 11
Performance evaluations of the methods for 3% contaminated IEX data - validation set 2.

Models	IEX2009		IEX2010		IEX2011		IEX2012	
	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate
SC93 [82]	10857.74	266.61%	43383.27	1322.27%	15095.29	1335.72%	20988.75	225.32%
C96 [23]	1673.6	10.79%	1013.33	**0.00%***	59234.15	5533.78%	14948.15	1090.86%
H01 [57][1]	19876.51	2040.09%	20064.23	2155.60%	18809.41	1581.47%	12436.68	1868.20%
H01 [57][2]	20376.48	2142.55%	31914.36	3378.52%	38429.03	3548.44%	16444.09	2633.44%
HY06 [59]	914.42	6.54%	962.9	5.15%	53782.8	5307.81%	643.81	5.61%
A09 [2]	1020.84	3.52%	1091.49	17.65%	1167.07	19.40%	1153.89	19.79%
Y13 [98]	1057.53	0.90%	1253.84	10.31%	1110.33	15.65%	1905.81	17.54%
E15 [13]	1346.4	47.75%	1543.45	50.73%	1484.4	46.11%	720.44	7.94%
C-R-FTSM	**808.94**	**0.03%***	**907.42**	4.10%	**861.47**	**1.62%***	**579.69**	**0.79%***

Table 12
Performance evaluations of the methods for 5% contaminated IEX data - validation set 1.

Models	IEX2009		IEX2010		IEX2011		IEX2012	
	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate
SC93 [82]	18571.13	1705.48%	25136.53	2261.28%	9997.5	804.48%	18314.23	326.99%
C96 [23]	97227.38	9352.40%	76226.63	7060.59%	67084.34	5967.30%	991.16	6.48%
H01 [57][1]	282415.66	27184.69%	265024.36	25687.10%	170677.31	15215.21%	2802	318.63%
H01 [57][2]	284082.28	33894.96%	335665.74	24784.41%	414242.31	41952.07%	5541.56	698.81%
HY06 [59]	52910.08	5842.95%	13205.42	1083.92%	26064.35	2556.51%	785.71	20.09%
A09 [2]	1571.16	121.96%	1787.19	62.95%	1173.84	26.25%	1000.56	22.68%
Y13 [98]	956.27	17.90%	1444.19	46.16%	1107.48	23.24%	1498.67	62.64%
E15 [13]	1416.07	70.05%	1484.78	38.68%	1100.55	13.65%	829.67	18.61%
C-R-FTSM	**683.06**	**4.63%***	**928.33**	**3.01%***	**832.52**	**0.88%***	**620.85**	**2.27%***

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Table 13
Performance evaluations of the methods for 5% contaminated IEX data - validation set 2.

Models	IEX2009		IEX2010		IEX2011		IEX2012	
	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate	RMSE	Variance Rate
SC93 [82]	10857.74	333.16%	43383.27	335.37%	15095.29	1335.72%	20988.75	462.73%
C96 [23]	1673.6	132.60%	1013.33	23.23%	59234.15	5533.78%	14948.15	75.39%
H01 [57][1]	19876.51	28119.71%	20064.231	27900.21%	18809.41	1581.47%	12436.683	23715.83%
H01 [57][2]	20376.48	28928.62%	31914.36	36671.54%	38429.038	3548.44%	16444.099	29798.20%
HY06 [59]	914.42	7.35%	962.9	**4.21%***	53782.8	5307.81%	643.81	3.75%
A09 [2]	1020.84	41.92%	1091.49	58.74%	1167.07	19.40%	1153.89	43.84%
Y13 [98]	1057.53	10.07%	1253.84	16.72%	1110.33	15.65%	1905.81	13.32%
E15 [13]	1346.4	57.74%	1543.45	42.03%	1484.4	46.11%	720.44	113.96%
C-R-FTSM	**808.94**	**1.46%***	**907.42**	5.44%	**861.47**	**1.62%***	**579.69**	**0.37%***

In the light of Tables 10-13, we can obviously notice that the proposed C-R-FTSM is the model that is fewest affected by outliers in case of both 3% and 5% contamination rates apart from IEX-2010 data for validation set 2. Even these cases, when it is compared with the analysis of original data, the results of proposed model show admissible change with around 5% variation rates. Moreover the combined model has the best forecasting performance for whole data and cases. Taking into account all of these information we can say that the proposed C-R-FTSM displays remarkable forecasting performance as well as having quite strong robustness characteristic.

In other respects, we sum up some results including the average variation rates for whole cases in Table 14. From these summarized results, it is noticed that the proposed robust forecasting model, for whole data sets, is almost never influenced by outliers with 3% and less than 3% average variation rates in comparison with the original data analysis. The average variation rates of the models which have the lowest four values are visually represented in Fig. 18.

Table 14
The average variation rates for whole IEX data.

Models	Contaminated Rate			
	3%		5%	
	Validation set 1	Validation set 2	Validation set 1	Validation set 2
SC93 [82]	1110.90%	787.48%	1274.56%	742.89%
C96 [23]	4796.64%	1658.86%	5596.69%	1876.24%
H01 [57][1]	15398.22%	1911.34%	17101.41%	25954.44%
H01 [57][2]	25553.17%	2925.74%	25332.56%	34073.08%
HY06 [59]	2510.15%	1331.28%	2375.87%	1350.26%
A09 [2]	50.69%	15.09%	58.46%	57.98%
Y13 [98]	14.74%	11.10%	37.49%	15.24%
E15 [13]	30.41%	38.13%	35.25%	63.71%
The C-R-FTSM	**3.19%**	**1.63%**	**2.70%**	**2.08%**

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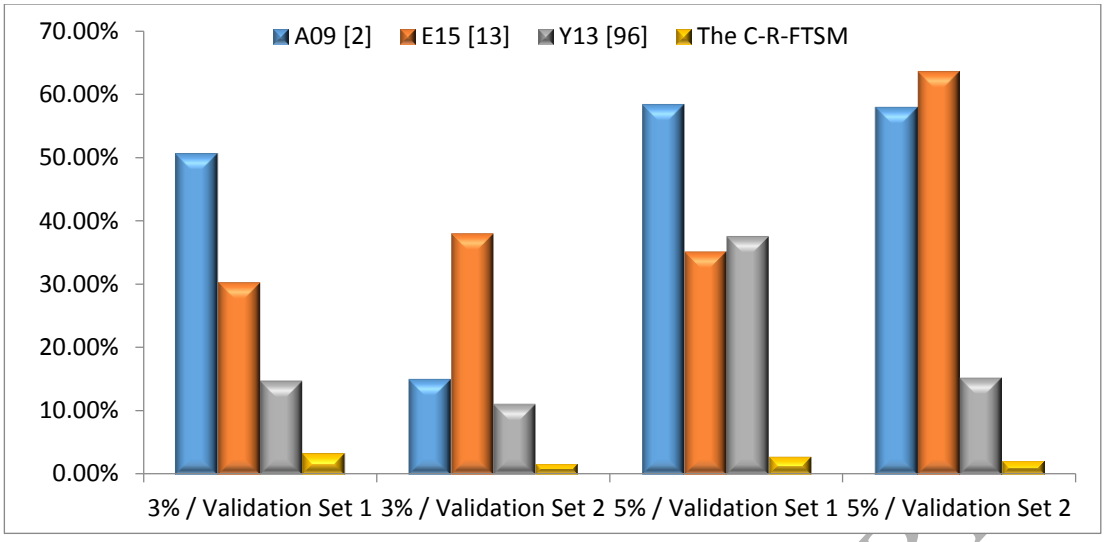


Fig. 18. The average variation rates of the models having the four lowest values - IEX data

6. Conclusions and future work

In this paper, a robust fuzzy time series forecasting model based upon combination of single multiplicative neuron model, particle swarm optimization and M-estimators has been introduced to improve prediction accuracy and to reveal the behaviour of the models in case of outliers. It must be emphasized that the proposed model has three remarkable characteristics absent in the current FTS models as being a first fuzzy time series model that has robustness in the FTS literature, fulfilling the analysis process in a single procedure to reduce modeling error that may occur in each stage of analysis and using the fuzzy inputs composed of the membership values in conjunction with the crisp data in just the same way as fuzzy inference systems. The various implementations, in total 30 analysis, have been carried out to prove effectiveness and robustness of the proposed combined robust FTS forecasting model by using TAIEX and IEX data sets. The data sets artificially contaminated with outliers at the rate of 3 and 5 percent have been used in the implementations as well as original data sets. Two different setups for each data set have been investigated with different sizes of training and validation data sets. It can be concluded from this study that the introduced C-R-FTSM has the best ability of prediction and robustness for almost all cases and data sets. Even when C-R-FTSM comes off second-best, it is competitive with the other models in the literature.

In consideration of the various implementations' results and the mentioned characteristics of the introduced C-R-FTSM, it is certainly said that it can comprehend almost all kinds of time series and perform the remarkable prediction performance even if they contain outlier(s). All

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of these constitute the motivation to apply the C-R-FTSM in the wide practical area of time series prediction problems.

A further step of this study may investigate the influence of different robust objective functions on forecasting performance and robustness of the model. Moreover, another future work may focus on coming up with an idea of new robust fuzzy inference system for time series prediction.

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