MATHEMATICA MONTISNIGRI
Vol LVII (2023)

APPLICATIONS OF JACOBSTHAL AND JACOBSTHAL-LUCAS NUMBERS IN CODING THEORY

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DOI: 10.20948/mathmontis-2023-57-4

Summary. In this article, we have developed a new method for coding\decoding the Jacobsthal and Jacobsthal-Lucas sequences via matrix representations. In this method, coding\decoding is done by transforming the messages into a square matrix. This process aims to not only increase the reliability of information security technology but also to provide the ability to verify information at a high rate.

1 INTRODUCTION

The sequences of Jacobsthal and Jacosthal-Lucas numbers are defined by

$$J_{n+2} = J_{n+1} + 2J_n, n \geq 0$$

with $J_0 = 0$ and $J_1 = 1$.

$$j_{n+2} = j_{n+1} + 2j_n, n \geq 0$$

with $j_0 = 2$ and $j_1 = 1$ [1].

The Jacobsthal matrix F_n is defined in [2] as follows.

$$F_n = \begin{bmatrix} J_{n+1} & 2J_n \\ J_n & 2J_{n-1} \end{bmatrix}$$

The Jacobsthal-Lucas matrix S^n is defined in [2] as follows.

$$S^n = \begin{Bmatrix} 3^{n-1}\begin{bmatrix} j_{n+1} & 2j_n \\ j_n & 2j_{n-1} \end{bmatrix}, n \text{ is odd} \\ 3^n\begin{bmatrix} J_{n+1} & 2J_n \\ J_n & 2J_{n-1} \end{bmatrix}, n \text{ is even} \end{Bmatrix}.$$

In this study, we will take S^n as $S^n = \begin{bmatrix} j_{n+1} & j_n \\ j_n & j_{n-1} \end{bmatrix}$ in terms of using Jacobsthal-Lucas numbers.

Determinants of the Jacobsthal F_n and Jacosthal-Lucas matrix S^n are

$$Det(F_n) = J_{n+1}J_{n-1} - J_n^2 = (-1)^n 2^{n-1},$$
$$Det(S^n) = j_{n+1}j_{n-1} - j_n^2 = 9.(-2)^{n-1},$$

2020 Mathematics Subject Classification: 68P30,11B39, 11B37.
Key words and Phrases: Coding\decoding algorithm; Jacobsthal matrix; Jacobsthal-Lucas matrix; Minesweeper.

respectively.

Coding\decoding is a computer language formation used in the creation of different applications and websites, especially computer software. Today, many platforms such as internet browsers, operating systems, applications on mobile phones, and websites, which are used without any problems in homes and workplaces, are made with the help of coding.

Today, coding education, which is included in the kindergarten and primary school curricula in many developed countries, is the most important indicator that the world of the future will be built on coding and software. For this reason, the basis of keeping up with the technology and information age is to provide training on how coding is done and in which languages.

Coding theory has been studied by many authors with different number sequences [3-6]. In particular, the Fibonacci and Lucas sequences and their more general versions, the Fibonacci and Lucas p-number sequences, have been studied [7-13].

Taş et al. a new coding/decoding algorithm were developed using Fibonacci Q-matrices [14]. The main idea of this method is to divide message matrix into the block matrices of size 2×2. Hashemi et al. created a new code system on k-Fibonacci, which is the generalization of Fibonacci numbers [15]. Aşçi et al., Dişkaya et al. has brought a different perspective to coding by working on AES-like encryption [16,17]. Sundarayya et al. also discussed a general coding study on sequences of numbers similar to Fibonacci numbers but with different recurrences [18]. The "Minesweeper Model" method, which is based on the principle of bringing together different number sequences of coding and creating a stronger coding system, was handled by Uçar and brought a different perspective to coding algorithm [19].

There are also many encryption methods that use polynomials in addition to number sequence in [20,16].

In addition to having a 2nd degree characteristic equation, we can also say that a different coding study has been carried out on the Padovan number sequence, which will make encryption stronger with the 3rd degree equation [21].

In this paper, we studied the coding\decoding application on different number sequences using the Jacobsthal and Jacobsthal-Lucas number sequences. In addition, it is aimed to both increases the reliability and transfer the information correctly by using two different sequences in addition to the single number sequence.

2 A NEW CODING/DECODING METHOD

Now, let's give our coding method. Assume that matrices B_k, E_k, F_n and S^n are of the following form:

$$B_k = \begin{bmatrix} b_1^k & b_2^k \\ b_3^k & b_4^k \end{bmatrix}, E_k = \begin{bmatrix} e_1^k & e_2^k \\ e_3^k & e_4^k \end{bmatrix}, F_n = \begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix} \text{ and } S^n = \begin{bmatrix} s_1 & s_2 \\ s_3 & s_4 \end{bmatrix}.$$

Let's denote b the number of the block matrices B_k. For b, we choose the number n as follows.

$$n = \begin{cases} b, & b \leq 3 \\ \left[\!\left[\frac{b}{2}\right]\!\right], & b > 3 \end{cases}$$

Let's write the following character table according to $mod30$ for n. We begin the n for the first character in Table 1.

A	B	C	D	E	F	G	H	I	J
n	n+1	n+2	n+3	n+4	n+5	n+6	n+7	n+8	n+9
K	L	M	N	O	P	Q	R	S	T
n+10	n+11	n+12	n+13	n+14	n+15	n+16	n+17	n+18	n+19
U	V	W	X	Y	Z	*	.		
n+20	n+21	n+22	n+23	n+24	n+25	n+26	n+27		

Table 1. The first character

Now, let's write the following new coding/decoding algorithms.

Jacobsthal Blocking Algorithm

Coding:

Step 1. Split the matrix M into blocks $B_k (1 \leq k \leq m^2)$

Step 2. Select *n*.

Step 3. Determine b_j^k $(1 \leq j \leq 4)$

Step 4. Calculate $\det(B_k) \rightarrow d_k$.

Step 5. Install $R = \left[d_k, b_j^k\right]_{j \in \{1,2,4\}}$

Step 6. Finishing the algorithm.

Decoding:

Step 1. Calculate F_n

Step 2. Determine p_k $(1 \leq k \leq 4)$

Step 3. Calculate $p_1 b_1^k + p_3 b_2^k \rightarrow e_1^k, (1 \leq k \leq m^2)$

Step 4. Calculate $p_2 b_1^k + p_4 b_2^k \rightarrow e_2^k$

Step 5. Figure out $(-1)^n 2^{n-1} d_k = e_1^k\left(p_2 x_k + p_4 b_4^k\right) - e_2^k (p_1 x_k + p_3 b_4^k)$

Step 6. Substitute for $x_k = b_3^k$

Step 7. Install B_k

Step 8. Install M.

Step 9. Finishing the algorithm.

For $b > 3$, let's give the following example as an application of the algorithm.

Example 2.1 Assume that we have the message matrix for the following message text:

HAVE NICE DAY.

So, thus the following matrix M is obtained,

$$M = \begin{pmatrix} H & A & V & E \\ * & N & I & C \\ E & * & D & A \\ Y & . & * & * \end{pmatrix}.$$

Coding Algorithm:

Step 1. We can divide the message matrix M into the matrices $B_k, (1 \leq k \leq 4)$,

$$B_1 = \begin{pmatrix} H & A \\ * & N \end{pmatrix},\ B_2 = \begin{pmatrix} V & E \\ I & C \end{pmatrix},\ B_3 = \begin{pmatrix} E & * \\ Y & . \end{pmatrix} \text{ and } B_4 = \begin{pmatrix} D & A \\ * & * \end{pmatrix}.$$

Step 2. Since $b = 4 > 3$, we obtain $n = \left[\left[\frac{b}{2}\right]\right] = 2$. For $n = 2$ and the message matrix M, we use the following "character" in Table 2.

H	A	V	E	N	I	C	D
9	2	23	6	15	10	4	5
Y	.	*					
26	29	28					

Table 2. The characters for $n = 2$.

Step 3. We get the elements of the blocks $B_k, (1 \leq k \leq 4)$ in Table 3.

$b_1^1 = 9$	$b_2^1 = 2$	$b_3^1 = 28$	$b_4^1 = 15$
$b_1^2 = 23$	$b_2^2 = 6$	$b_3^2 = 10$	$b_4^2 = 4$
$b_1^3 = 6$	$b_2^3 = 28$	$b_3^3 = 26$	$b_4^3 = 29$
$b_1^5 = 5$	$b_2^4 = 2$	$b_3^4 = 28$	$b_4^4 = 28$

Table 3. Some elements of the blocks.

Step 4. Let's compute the determinants d_k of the blocks B_k:

$$d_1 = \det(B_1) = 79$$
$$d_2 = \det(B_2) = 32$$
$$d_3 = \det(B_3) = -554$$
$$d_4 = \det(B_4) = 84$$

Step 5. Using step 3 and step 4, we obtain the following matrix R

$$R = \begin{pmatrix} 79 & 9 & 2 & 15 \\ 32 & 23 & 6 & 4 \\ -554 & 6 & 28 & 29 \\ 84 & 5 & 2 & 28 \end{pmatrix}.$$

Step 6. Finishing the algorithm.

Decoding algorithm:

Step 1. It is known that for $n = 2$

$$F_2 = \begin{pmatrix} J_3 & 2J_2 \\ J_2 & 2J_1 \end{pmatrix} = \begin{pmatrix} 3 & 2.1 \\ 1 & 2.1 \end{pmatrix}$$

Step 2. The elements of F_2 are denoted by $p_1 = 3, p_2 = 1, p_3 = 1, p_4 = 1$

Step 3. We calculate the elements e_1^k to construct matrix E_k:

$$e_1^1 = p_1 b_1^1 + p_3 b_2^1 = 29$$
$$e_1^2 = p_1 b_1^2 + p_3 b_2^2 = 75$$
$$e_1^3 = p_1 b_1^3 + p_3 b_2^3 = 46$$
$$e_1^4 = p_1 b_1^4 + p_3 b_2^4 = 17$$

Similarly,

Step 4.

$$e_2^1 = p_2 b_1^1 + p_4 b_2^1 = 11$$
$$e_2^2 = p_2 b_1^2 + p_4 b_2^2 = 29$$
$$e_2^3 = p_2 b_1^3 + p_4 b_2^3 = 34$$
$$e_2^4 = p_2 b_1^4 + p_4 b_2^4 = 7$$

Step 5. We calculate the elements x_k:

$$(-1)^2(2)(79) = 29\big(1x_1 + 1(15)\big) - 11(3x_1 + 1(15))$$
$$x_1 = 28$$
$$(-1)^2(2)(32) = 75(1x_2 + 1(4)) - 29(3x_2 + 1(4))$$
$$x_2 = 10$$
$$(-1)^2(2)(-554) = 46(1x_3 + 1(29)) - 34(3x_3 + 1(29))$$
$$x_3 = 26$$
$$(-1)^2(2)(84) = 17(1x_4 + 1(28)) - 7(3x_4 + 1(28))$$
$$x_4 = 28$$

Step 6. We rename x_k as follows:

$$x_1 = b_3^1 = 28, x_2 = b_3^2 = 10, x_3 = b_3^3 = 26, x_4 = b_3^4 = 28$$

Step 7. We install the block matrices B_k:

$$B_1 = \begin{pmatrix} 9 & 2 \\ 28 & 15 \end{pmatrix}, B_2 = \begin{pmatrix} 23 & 6 \\ 10 & 4 \end{pmatrix}, B_3 = \begin{pmatrix} 6 & 28 \\ 26 & 29 \end{pmatrix}, B_4 = \begin{pmatrix} 5 & 2 \\ 28 & 28 \end{pmatrix}$$

Step 8. We obtain the message matrix:

$$M = \begin{pmatrix} 9 & 9 & 23 & 6 \\ 28 & 15 & 10 & 4 \\ 6 & 28 & 5 & 2 \\ 26 & 29 & 28 & 28 \end{pmatrix}$$

Step 9. Finishing the algorithm.

3 A MIXED MODEL: MINESWEEPER MODEL

Now, we give Minesweeper Model using Jacobsthal and Jacobsthal -Lucas matrix. The main purpose of this section is to increase the reliability of the decoding process by using the block matrix representations of both Jacobsthal and Jacobsthal -Lucas sequences in odd and even indices, respectively, during the decoding phase.

Coding algorithm:

Step 1. Split the matrix M into blocks $B_k, 1 \leq k \leq m^2$.

Step 2. Select n.

Step 3. Determine $b_j^k, 1 \leq j \leq 4$.

Step 4. Calculate $\det(B_k) \rightarrow d_k$.

Step 5. Install $R = \left[d_k, b_j^k\right]_{j\in\{1,2,3\}}$

Step 6. Finishing the algorithm.

Decoding algorithm:

Step 1. Calculate F_n

Step 2. Calculate S^n

Step 3. Calculate $p_1 b_1^k + p_3 b_2^k \rightarrow e_1^k, 0 \leq k-1 \leq 4m$

Step 4. Calculate $s_1 b_1^k + s_3 b_2^k \rightarrow e_1^k, 2 \leq k \leq 4m$

Step 5. Calculate $p_2 b_1^k + p_4 b_2^k \rightarrow e_2^k, 0 \leq k-1 \leq 4m$

Step 6. Calculate $s_2 b_1^k + s_4 b_2^k \rightarrow e_2^k, 2 \leq k \leq 4m$

Step 7. Figure out $(-1)^n 2^{n-1} d_k = e_1^k\left(p_2 b_3^k + p_4 x_k\right) - e_2^k\left(p_1 b_3^k + p_3 x_k\right), 0 \leq k-1 \leq 4m$

Step 8. Figure out $9(-2)^{n-1} d_k = e_1^k\left(s_2 b_3^k + s_4 x_k\right) - e_2^k\left(s_1 b_3^k + s_3 x_k\right), 2 \leq k \leq 4m$

Step 9. Substitute for $x_k = b_4^k$.

Step 10. Install B_k

Step 11. Install M

Step 12. Finishing the algorithm.

For this algorithm, let's give an application for $b > 3$.

Example 3.1 For the following message text. Let's assume that the message matrix:

"BEAUTIFUL MINDS INSPIRE OTHER"

So, we obtain matrix M:

$$M = \begin{pmatrix} B & E & A & U & T & I \\ F & U & L & * & M & I \\ N & D & S & * & I & N \\ S & P & I & R & E & * \\ O & T & H & E & R & * \\ * & * & * & * & * & * \end{pmatrix}$$

Coding algorithm:

Step 1. We can divide the message matrix M into the B_k.

$$B_1 = \begin{pmatrix} B & E \\ F & U \end{pmatrix}, B_2 = \begin{pmatrix} A & U \\ L & * \end{pmatrix}, B_3 = \begin{pmatrix} T & I \\ M & I \end{pmatrix}$$
$$B_4 = \begin{pmatrix} N & D \\ S & P \end{pmatrix}, B_5 = \begin{pmatrix} S & * \\ I & R \end{pmatrix}, B_6 = \begin{pmatrix} I & N \\ E & * \end{pmatrix}$$
$$B_7 = \begin{pmatrix} O & T \\ * & * \end{pmatrix}, B_8 = \begin{pmatrix} H & E \\ * & * \end{pmatrix}, B_9 = \begin{pmatrix} R & * \\ * & * \end{pmatrix}$$

Step 2. Since $b = 9 > 3$, we get $n = \left[\left[\frac{b}{2}\right]\right] = 4$. We use Table 4 according to $mod30$ for M matrix for $n = 4$.

B	E	A	U	T	I
5	8	4	24	23	12
F	L	M	N	D	S
9	15	16	17	7	22
P	R	O	H	*	
19	21	18	11	0	

Table 4. The characters for $n = 4$.

Step 3. We get the elements of the blocks B_k in Table 5.

$b_1^1 = 5$	$b_2^1 = 8$	$b_3^1 = 4$	$b_4^1 = 24$
$b_1^2 = 4$	$b_2^2 = 24$	$b_3^2 = 15$	$b_4^2 = 0$
$b_1^3 = 23$	$b_2^3 = 12$	$b_3^3 = 16$	$b_4^3 = 12$
$b_1^4 = 17$	$b_2^4 = 7$	$b_3^4 = 22$	$b_4^4 = 19$
$b_1^5 = 22$	$b_2^5 = 0$	$b_3^5 = 12$	$b_4^5 = 21$
$b_1^6 = 12$	$b_2^6 = 17$	$b_3^6 = 8$	$b_4^6 = 0$
$b_1^7 = 18$	$b_2^7 = 23$	$b_3^7 = 0$	$b_4^7 = 0$
$b_1^8 = 11$	$b_2^8 = 8$	$b_3^8 = 0$	$b_4^8 = 0$
$b_1^9 = 21$	$b_2^9 = 0$	$b_3^9 = 0$	$b_4^9 = 0$

Table 5. Some elements of the blocks.

Step 4. We obtain the determinants d_k of the blocks B_k:

$$d_1 = \det(B_1) = 88, d_2 = \det(B_2) = -360, d_3 = \det(B_3) = 84$$
$$d_4 = \det(B_4) = 169, d_5 = \det(B_5) = 462, d_6 = \det(B_6) = -136$$
$$d_7 = \det(B_7) = 0, d_8 = \det(B_8) = 0, d_9 = \det(B_9) = 0$$

Step 5. From step 3 and step 4, we find the following matrix:

$$R = \begin{pmatrix} 88 & 5 & 8 & 4 \\ -360 & 4 & 24 & 15 \\ 84 & 23 & 12 & 16 \\ 169 & 17 & 7 & 22 \\ 462 & 22 & 0 & 12 \\ -136 & 12 & 17 & 8 \\ 0 & 18 & 23 & 0 \\ 0 & 11 & 8 & 0 \\ 0 & 21 & 0 & 0 \end{pmatrix}.$$

Step 6. Finishing the algorithm.

Decoding algorithm:

Step 1. For $n = 4$,

$$F_4 = \begin{pmatrix} 11 & 2.5 \\ 5 & 2.3 \end{pmatrix}$$

Step 2.

$$S^4 = \begin{pmatrix} 31 & 17 \\ 17 & 7 \end{pmatrix}$$

Step 3. If k is an odd number, then we use Jacobsthal F-matrix to find the elements $e_1^k, k = 1,3,5,7,9$ for the matrix E_k;

$$e_1^1 = 95, e_1^3 = 313, e_1^5 = 242, e_1^7 = 313, e_1^9 = 231$$

Step 4. If k is an even number, then we use Jacobsthal-Lucas S-matrix to find the elements $e_1^k, k = 2,4,6,8$ for the matrix E_k;

$$e_1^2 = 532, e_1^4 = 646, e_1^6 = 661, e_1^8 = 477$$

Step 5. If k is an odd number, then we use Jacobsthal F-matrix to find the elements $e_2^k, k = 1,3,5,7,9$ for the matrix E_k;

$$e_2^1 = 49, e_2^3 = 151, e_2^5 = 110, e_2^7 = 159, e_2^9 = 105$$

Step 6. If k is an even number, then we use Jacobsthal-Lucas S-matrix to find the elements $e_2^k, k = 2,4,6,8$ for the matrix E_k;

$$e_2^2 = 236, e_2^4 = 338, e_2^6 = 323, e_2^8 = 243$$

Step 7. If k is an odd number, then we use Jacobsthal F-matrix to find the elements $x_k, k = 1,3,5,7,9$.

$$(-1)^4 2^3.88 = 95(5.4 + 3x_1) - 49(11.4 + 5x_1) \Longrightarrow x_1 = 24,$$
$$(-1)^4 2^3.84 = 313(5.16 + 3x_3) - 151(11.16 + 5x_3) \Longrightarrow x_3 = 12,$$
$$(-1)^4 2^3.462 = 242(5.12 + 3x_5) - 110(11.12 + 5x_5) \Longrightarrow x_5 = 21,$$
$$(-1)^4 2^3.0 = 313(5.0 + 3x_7) - 159(11.0 + 5x_7) \Longrightarrow x_7 = 0,$$
$$(-1)^4 2^3.0 = 231(5.0 + 3x_9) - 105(11.0 + 5x_9) \Longrightarrow x_9 = 0.$$

Step 8. If k is an even number, then we use Jacobsthal-Lucas S-matrix to find the elements $x_k, k = 2,4,6,8.$

$$-9.(-2)^3 360 = 532(17.15 + 7x_2) - 236(31.15 + 17x_2) \Longrightarrow x_2 = 0,$$
$$9.(-2)^3 169 = 646(17.22 + 7x_4) - 338(31.22 + 17x_4) \Longrightarrow x_4 = 19,$$
$$-9.(-2)^3 136 = 661(17.8 + 7x_6) - 323(31.8 + 17x_6) \Longrightarrow x_6 = 0,$$
$$9.(-2)^3 0 = 477(17.0 + 7x_8) - 243(31.0 + 17x_8) \Longrightarrow x_8 = 0.$$

Step 9. We rename x_k as follows:

$$x_1 = b_4^1 = 24, x_2 = b_4^2 = 0, x_3 = b_4^3 = 12,$$
$$x_4 = b_4^4 = 19, x_5 = b_4^5 = 21, x_6 = b_4^6 = 0,$$
$$x_7 = b_4^7 = 0, x_8 = b_4^8 = 0, x_9 = b_4^9 = 0.$$

Step 10. We get the block matrices B_k:

$$B_1 = \begin{pmatrix} 5 & 8 \\ 9 & 24 \end{pmatrix}, B_2 = \begin{pmatrix} 4 & 24 \\ 15 & 0 \end{pmatrix}, B_3 = \begin{pmatrix} 23 & 12 \\ 16 & 12 \end{pmatrix},$$
$$B_4 = \begin{pmatrix} 17 & 7 \\ 22 & 19 \end{pmatrix}, B_5 = \begin{pmatrix} 22 & 0 \\ 12 & 21 \end{pmatrix}, B_6 = \begin{pmatrix} 12 & 17 \\ 8 & 0 \end{pmatrix},$$
$$B_7 = \begin{pmatrix} 18 & 23 \\ 0 & 0 \end{pmatrix}, B_8 = \begin{pmatrix} 11 & 8 \\ 0 & 0 \end{pmatrix}, B_9 = \begin{pmatrix} 21 & 0 \\ 0 & 0 \end{pmatrix}.$$

Step 11. We obtain the following message matrix M:

$$M = \begin{pmatrix} 5 & 8 & 4 & 24 & 23 & 12 \\ 9 & 24 & 15 & 0 & 16 & 12 \\ 17 & 7 & 22 & 0 & 12 & 17 \\ 22 & 19 & 12 & 21 & 8 & 0 \\ 18 & 23 & 11 & 8 & 21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} B & E & A & U & T & I \\ F & U & L & * & M & I \\ N & D & S & * & I & N \\ S & P & I & R & E & * \\ O & T & H & E & R & * \\ * & * & * & * & * & * \end{pmatrix}$$

Step 12. Finishing the algorithm.

4 CONCLUSIONS

In this paper, we studied the coding\decoding application on different number sequences using the Jacobsthal and Jacobsthal-Lucas number sequences. In addition, it is aimed to both increases the reliability and transfer the information correctly by using two different

sequences in addition to the single number sequence. Different security coding\decoding can be created by applying this work to all number sequences that require binary recurrence.

Acknowledgements: We would like to thank the editor and referees for their valuable comments, and remarks that led to a great improvement of the article.

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Received, June 12, 2022