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Novel Distance Measure for Linear Diophantine Fuzzy Sets

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Abstract

Unlike other fuzzy set theories, the theory of Linear Diophantine fuzzy set (LDFS) aims to overcome uncertainties via the reference parameters. The Parameters provide flexibility and efficiency to LDFS in handling the uncertain data. In this study, we introduce new distance measure for linear Diophantine fuzzy sets. Then, we studied the properties of proposed distance measure. Finally, we established a new similarity measure for LDFSs based on distance measure.

AMS Subject Classification: 03E72, 28E10, 47S40

Keywords: Distance measure, linear diophantine fuzzy sets, q-ROFS, IFS, PyFSs

INTRODUCTION

Since the theory of fuzzy set (FS) was established by Zadeh [1], many researcher proposed new approaches and theories to overcome the uncertainties and vagueness. Firstly, Atanassov [2] introduced intuitionistic fuzzy set (IFS) as an extension of FS with the concept of membership degree and non-membership degree with the sum of them between 0 and 1. Yager [3] introduced Pythagorean fuzzy set (PyFS) by using the condition that the sum of the squares of the membership and non-member degrees is equal or less than 1. Yager [4] proposed q-rung orthopair fuzzy set (q-ROFS) as an extension of PyFS with the condition of sum of q^{th} (where $q \geq 1$) power of membership and non-membership degrees between 0 and 1. This means that if $q = 1$, then q-ROFS is reduced to IFS and if $q = 2$, then q-ROFS is induced to PFS. Recently, Riaz and Hashmi [5] proposed the concept of linear Diophantine fuzzy set (LDFS) with the addition of reference parameters to remove the limitations on membership and non-membership degrees of fuzzy sets.

As is well known, distance measure is an important mathematical tool in fuzzy set theory, which is used to distinguish between two fuzzy sets. Many researchers have established distance measures between fuzzy sets. For example, Atanassov [6] introduced distance measures between the IFSs. Burillo and Bustince [7] proposed the normalized Hamming distance and normalized Euclidean distance between the IFSs. Grzegorzewski [8] gave new definitions of Hamming and Euclidean distance measures on IFSs based on Hausdorff metric. Wang and Xin [9] gave the axiomatic definition of distance measures on IFSs and introduced new distance measures between the IFSs. Ngan et al. [10] defined H-max distance measure for IFSs. Peng et al. [11] introduced several distance measures for PyFSs. Li and Zeng [12] proposed distance measure for PFSs, which take into account the parameters of strength and direction of commitment. Li and Lu [13] gave generalization of Hamming and Hausdorff distance measures for PyFSs. Mahanta and Panda [14] introduced nee distance measure for PyFSs. Du [15] presented

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Minkowski-type distance measures for q-ROFSs. Peng and Liu [16] proposed several distance measures for PyFSs. In LDFS, Mohammad et al. [17] and Gül and Aydoğdu [18] introduced Euclidean and Hamming distance measures, simultaneously and unknowingly. Also, Mohammad et al. [17] gave the generalization of Euclidean and Hamming distance measures for LDFS.

The rest of this paper is organized as follows. Section 2 explains the preliminaries of LDFS. In Section 3, new distance measure for LDFS is given and its properties are exhibited. Then, a new similarity measure based on proposed distance measure for LDFSs. In section 4, the conclusion of this work is given.

PRELIMINARIES

In this section, we give fundamental definitions and some properties of LDFSs.

Linear Diophantine Fuzzy Sets

Riaz and Hashmi [5] proposed the concept of linear Diophantine fuzzy set (LDFS) with the addition of reference parameters to remove the limitations on membership and non-membership grades of IFSs, PFSs, and q-ROFSs and gave fundamental properties of LDFS as follow:

Definition 2.1: Let $\mathfrak{U}$ be the universal set. An LDFS $\mathfrak{D}$ on $\mathfrak{U}$ is defined as

$$\mathfrak{D} = \{(\mathfrak{u}, \langle \mu_{\mathfrak{D}}(\mathfrak{u}), \nu_{\mathfrak{D}}(\mathfrak{u})\rangle, \langle \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u})\rangle) : \mathfrak{u} \in \mathfrak{U}\} \quad (1)$$

where $\mu_{\mathfrak{D}}(\mathfrak{u}), \nu_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u}) \in [0,1]$ are membership grade, non-membership grade, and reference parameters, respectively. These grades and reference parameters satisfy the following conditions: $0 \leq \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}) + \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u}) \leq 1$ and $0 \leq \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u})\mu_{\mathfrak{D}}(\mathfrak{u}) + \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u})\nu_{\mathfrak{D}}(\mathfrak{u}) \leq 1$ for all $\mathfrak{u} \in \mathfrak{U}$.

The indeterminacy degree, $\eta_{\mathfrak{D}}$, is evaluated as $c_{\mathfrak{D}}(\mathfrak{u})\eta_{\mathfrak{L}}(\mathfrak{u}) = 1 - (\mathfrak{a}_{\mathfrak{D}}(\mathfrak{u})\mu_{\mathfrak{D}}(\mathfrak{u}) + \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u})\nu_{\mathfrak{D}}(\mathfrak{u}))$ where $c_{\mathfrak{D}}(\mathfrak{u})$ is the reference parameter of indeterminacy degree.

The absolute LDFS and null LDFS on $\mathfrak{U}$ are defined as follow, respectively:

$$\mathfrak{A} = \{(\mathfrak{u}, \langle 1,0\rangle, \langle 1,0\rangle) : \mathfrak{u} \in \mathfrak{U}\} \quad (2)$$

$$\mathfrak{N} = \{(\mathfrak{u}, \langle 0,1\rangle, \langle 0,1\rangle) : \mathfrak{u} \in \mathfrak{U}\} \quad (3)$$

Definition 2.2: A linear Diophantine fuzzy number (LDFN) is a collection of $\mathfrak{L} = (\langle \mu_{\mathfrak{L}}, \nu_{\mathfrak{L}}\rangle, \langle \mathfrak{a}_{\mathfrak{L}}, \mathfrak{b}_{\mathfrak{L}}\rangle)$ with the following conditions:

1. $\mu_{\mathfrak{L}}, \nu_{\mathfrak{L}}, \mathfrak{a}_{\mathfrak{L}}, \mathfrak{b}_{\mathfrak{L}} \in [0,1]$,
2. $0 \leq \mathfrak{a}_{\mathfrak{L}} + \mathfrak{b}_{\mathfrak{L}} \leq 1$,
3. $0 \leq \mathfrak{a}_{\mathfrak{L}}\mu_{\mathfrak{L}} + \mathfrak{b}_{\mathfrak{L}}\nu_{\mathfrak{L}} \leq 1$.

We denote by $LDFN(\mathfrak{U})$ all LDFNs on $\mathfrak{U}$.

Definition 2.3: Let $\mathfrak{D}\{(\mathfrak{u}, \langle \mu_{\mathfrak{D}}(\mathfrak{u}), \nu_{\mathfrak{D}}(\mathfrak{u})\rangle, \langle \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u})\rangle) : \mathfrak{u} \in \mathfrak{U}\}$ and $\mathfrak{E} = \{(\mathfrak{u}, \langle \mu_{\mathfrak{E}}(\mathfrak{u}), \nu_{\mathfrak{E}}(\mathfrak{u})\rangle, \langle \mathfrak{a}_{\mathfrak{E}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{E}}(\mathfrak{u})\rangle) : \mathfrak{u} \in \mathfrak{U}\}$ be two LDFSs on $\mathfrak{U}$, then

1. $\mathfrak{D}^c = \{(\mathfrak{u}, \langle \nu_{\mathfrak{D}}(\mathfrak{u}), \mu_{\mathfrak{D}}(\mathfrak{u})\rangle, \langle \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u})\rangle) : \mathfrak{u} \in \mathfrak{U}\}$
2. $\mathfrak{D} = \mathfrak{E}$ if and only if $\mu_{\mathfrak{D}}(\mathfrak{u}) = \mu_{\mathfrak{E}}(\mathfrak{u}), \nu_{\mathfrak{D}}(\mathfrak{u}) = \nu_{\mathfrak{E}}(\mathfrak{u}), \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}) = \mathfrak{a}_{\mathfrak{E}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u}) = \mathfrak{b}_{\mathfrak{E}}(\mathfrak{u})$
3. $\mathfrak{D} \subseteq \mathfrak{E}$ if and only if $\mu_{\mathfrak{D}}(\mathfrak{u}) \leq \mu_{\mathfrak{E}}(\mathfrak{u}), \nu_{\mathfrak{D}}(\mathfrak{u}) \geq \nu_{\mathfrak{E}}(\mathfrak{u}), \mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}) \leq \mathfrak{a}_{\mathfrak{E}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{D}}(\mathfrak{u}) \geq \mathfrak{b}_{\mathfrak{E}}(\mathfrak{u})$
4. $\mathfrak{D} \cup \mathfrak{E} = \left\{\left(\mathfrak{u}, \begin{matrix}\langle \max(\mu_{\mathfrak{D}}(\mathfrak{u}), \mu_{\mathfrak{E}}(\mathfrak{u})), \min(\nu_{\mathfrak{D}}(\mathfrak{u}), \nu_{\mathfrak{E}}(\mathfrak{u}))\rangle, \\ \langle \max(\mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{a}_{\mathfrak{E}}(\mathfrak{u})), \min(\mathfrak{b}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{E}}(\mathfrak{u}))\rangle\end{matrix}\right) : \mathfrak{u} \in \mathfrak{U}\right\}$
5. $\mathfrak{D} \cap \mathfrak{E} = \left\{\left(\mathfrak{u}, \begin{matrix}\langle \min(\mu_{\mathfrak{D}}(\mathfrak{u}), \mu_{\mathfrak{E}}(\mathfrak{u})), \max(\nu_{\mathfrak{D}}(\mathfrak{u}), \nu_{\mathfrak{E}}(\mathfrak{u}))\rangle, \\ \langle \min(\mathfrak{a}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{a}_{\mathfrak{E}}(\mathfrak{u})), \max(\mathfrak{b}_{\mathfrak{D}}(\mathfrak{u}), \mathfrak{b}_{\mathfrak{E}}(\mathfrak{u}))\rangle\end{matrix}\right) : \mathfrak{u} \in \mathfrak{U}\right\}$

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Definition 2.4: Let $\mathfrak{L}_1 = \left(\langle \mu_{\mathfrak{L}_1}, \nu_{\mathfrak{L}_1}\rangle, \langle \mathrm{a}_{\mathfrak{L}_1}, \mathrm{b}_{\mathfrak{L}_1}\rangle\right)$ and $\mathfrak{L}_2 = \left(\langle \mu_{\mathfrak{L}_2}, \nu_{\mathfrak{L}_2}\rangle, \langle \mathrm{a}_{\mathfrak{L}_2}, \mathrm{b}_{\mathfrak{L}_2}\rangle\right)$ be two LDFNs on $\mathfrak{U}$ and $\mathfrak{n} > 0$ then

1. $\mathfrak{L}_1 \oplus \mathfrak{L}_2 = \left(\langle \mu_{\mathfrak{L}_1} + \mu_{\mathfrak{L}_2} - \mu_{\mathfrak{L}_1}\mu_{\mathfrak{L}_2}, \nu_{\mathfrak{L}_1}\nu_{\mathfrak{L}_2}\rangle, \langle \mathrm{a}_{\mathfrak{L}_1} + \mathrm{a}_{\mathfrak{L}_2} - \mathrm{a}_{\mathfrak{L}_1}\mathrm{a}_{\mathfrak{L}_2}, \mathrm{b}_{\mathfrak{L}_1}\mathrm{b}_{\mathfrak{L}_2}\rangle\right)$
2. $\mathfrak{L}_1 \otimes \mathfrak{L}_2 = \left(\langle \mu_{\mathfrak{L}_1}\mu_{\mathfrak{L}_2}, \nu_{\mathfrak{L}_1} + \nu_{\mathfrak{L}_2} - \nu_{\mathfrak{L}_1}\nu_{\mathfrak{L}_2}\rangle, \langle \mathrm{a}_{\mathfrak{L}_1}\mathrm{a}_{\mathfrak{L}_2}, \mathrm{b}_{\mathfrak{L}_1} + \mathrm{b}_{\mathfrak{L}_2} - \mathrm{b}_{\mathfrak{L}_1}\mathrm{b}_{\mathfrak{L}_2}\rangle\right)$
3. $\mathfrak{n}\mathfrak{L}_1 = \left(\langle 1 - \left(1 - \mu_{\mathfrak{L}_1}\right)^{\mathfrak{n}}, \nu_{\mathfrak{L}_1}^{n}\rangle, \langle 1 - \left(1 - \mathrm{a}_{\mathfrak{L}_1}\right)^{\mathfrak{n}}, \mathrm{b}_{\mathfrak{L}_1}^{\mathfrak{n}}\rangle\right)$
4. $\mathfrak{L}_1^{\mathfrak{n}} = \left(\langle \mu_{\mathfrak{L}_1}^{n}, 1 - \left(1 - \nu_{\mathfrak{L}_1}\right)^{\mathfrak{n}}\rangle, \langle \mathrm{a}_{\mathfrak{L}_1}^{\mathfrak{n}}, 1 - \left(1 - \mathrm{b}_{\mathfrak{L}_1}\right)^{\mathfrak{n}}\rangle\right)$

Definition 2.5: Let $\mathfrak{L} = (\langle \mu_{\mathfrak{L}}, \nu_{\mathfrak{L}}\rangle, \langle \mathrm{a}_{\mathfrak{L}}, \mathrm{b}_{\mathfrak{L}}\rangle)$ be an LDFN. The mapping $s: LDFN(\mathfrak{U}) \to [-1,1]$ is called score function on LDFN $\mathfrak{L}$ defined by

$$s(\mathfrak{L}) = \frac{1}{2}[(\mu_{\mathfrak{L}} - \nu_{\mathfrak{L}}) + (\mathrm{a}_{\mathfrak{L}} - \mathrm{b}_{\mathfrak{L}})]$$

Definition 2.6: Let $\mathfrak{L} = (\langle \mu_{\mathfrak{L}}, \nu_{\mathfrak{L}}\rangle, \langle \mathrm{a}_{\mathfrak{L}}, \mathrm{b}_{\mathfrak{L}}\rangle)$ be an LDFN. The mapping $a: LDFN(\mathfrak{U}) \to [0,1]$ defined by

$$a(\mathfrak{L}) = \frac{1}{2}\left[\left(\frac{\mu_{\mathfrak{L}} + \nu_{\mathfrak{L}}}{2}\right) + (\mathrm{a}_{\mathfrak{L}} + \mathrm{b}_{\mathfrak{L}})\right]$$

is called accuracy function on $\mathfrak{L}$.

Mohammad et al. [17] introduced generalized distance measure between two LDFSs $\mathfrak{D}$ and $\mathfrak{E}$ as follow:

$$d_M(\mathfrak{D}, \mathfrak{E}) = \left(\frac{1}{4n}\sum_{i=1}^{n}\begin{bmatrix} |\mu_{\mathfrak{D}}(\mathrm{u}_i) - \mu_{\mathfrak{E}}(\mathrm{u}_i)|^p + |\nu_{\mathfrak{D}}(\mathrm{u}_i) - \nu_{\mathfrak{E}}(\mathrm{u}_i)|^p \\ +|\mathrm{a}_{\mathfrak{D}}(\mathrm{u}_i) - \mathrm{a}_{\mathfrak{E}}(\mathrm{u}_i)|^p + |\mathrm{b}_{\mathfrak{D}}(\mathrm{u}_i) - \mathrm{b}_{\mathfrak{E}}(\mathrm{u}_i)|^p \end{bmatrix}\right)^{\frac{1}{p}}$$

For $p = 1,2$ we get normalized Hamming distance and normalized Euclidean distance between two LDFSs $\mathcal{L}$ and $\mathcal{M}$ as follow, respectively:

$$d_H(\mathfrak{D}, \mathfrak{E}) = \frac{1}{4n}\sum_{i=1}^{n}\begin{bmatrix} |\mu_{\mathfrak{D}}(\mathrm{u}_i) - \mu_{\mathfrak{E}}(\mathrm{u}_i)| + |\nu_{\mathfrak{D}}(\mathrm{u}_i) - \nu_{\mathfrak{E}}(\mathrm{u}_i)| \\ +|\mathrm{a}_{\mathfrak{D}}(\mathrm{u}_i) - \mathrm{a}_{\mathfrak{E}}(\mathrm{u}_i)| + |\mathrm{b}_{\mathfrak{D}}(\mathrm{u}_i) - \mathrm{b}_{\mathfrak{E}}(\mathrm{u}_i)| \end{bmatrix},$$

$$d_E(\mathfrak{D}, \mathfrak{E}) = \left(\frac{1}{4n}\sum_{i=1}^{n}\begin{bmatrix} |\mu_{\mathfrak{D}}(\mathrm{u}_i) - \mu_{\mathfrak{E}}(\mathrm{u}_i)|^2 + |\nu_{\mathfrak{D}}(\mathrm{u}_i) - \nu_{\mathfrak{E}}(\mathrm{u}_i)|^2 \\ +|\mathrm{a}_{\mathfrak{D}}(\mathrm{u}_i) - \mathrm{a}_{\mathfrak{E}}(\mathrm{u}_i)|^2 + |\mathrm{b}_{\mathfrak{D}}(\mathrm{u}_i) - \mathrm{b}_{\mathfrak{E}}(\mathrm{u}_i)|^2 \end{bmatrix}\right)^{\frac{1}{2}}$$

MAIN RESULTS

In this section, we introduce a new distance measure for LDFSs and give its properties. Then, we obtained a new similarity measure for LDFSs based on distance measure.

Definition 3.1: Let $\mathfrak{D}$ and $\mathfrak{E}$ be two LDFSs. If the mapping $d: LDFS(\mathfrak{U}) \times LDFS(\mathfrak{U}) \to [0,1]$ satisfies the following properties, then d is called a distance measure between LDFSs $\mathfrak{D}$ and $\mathfrak{E}$.

D1) $0 \le d(\mathfrak{D}, \mathfrak{E}) \le 1$,

D2) $d(\mathfrak{D}, \mathfrak{E}) = 0$ if and only if $\mathfrak{D} = \mathfrak{E}$,

D3) $d(\mathfrak{D}, \mathfrak{E}) = d(\mathfrak{E}, \mathfrak{D})$,

D4) If $\mathfrak{D} \subseteq \mathfrak{E} \subseteq \mathfrak{F}$, then $d(\mathfrak{D}, \mathfrak{F}) \ge d(\mathfrak{D}, \mathfrak{E})$ and $d(\mathfrak{D}, \mathfrak{F}) \ge d(\mathfrak{E}, \mathfrak{F})$ for all $\mathfrak{D}, \mathfrak{E}, \mathfrak{F} \in LDFS(\mathfrak{U})$.

Let $\mathfrak{D}$ and $\mathfrak{E}$ be two LDFSs on universal set $\mathfrak{U}$. We define a mapping on $LDF(\mathfrak{U})$ as follows

$$d(\mathfrak{D}, \mathfrak{E}) = \frac{1}{2}\begin{bmatrix} \max_{\mathrm{u}\in\mathfrak{U}}(|\mathrm{a}_{\mathfrak{D}}(\mathrm{u})\mu_{\mathfrak{D}}(\mathrm{u}) - \mathrm{a}_{\mathfrak{E}}(\mathrm{u})\mu_{\mathfrak{E}}(\mathrm{u})|) \\ + \max_{\mathrm{u}\in\mathfrak{U}}(|\mathrm{b}_{\mathfrak{D}}(\mathrm{u})\nu_{\mathfrak{D}}(\mathrm{u}) - \mathrm{b}_{\mathfrak{E}}(\mathrm{u})\nu_{\mathfrak{E}}(\mathrm{u})|) \end{bmatrix}$$

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Theorem 3.2: Let $\mathfrak{D}$ and $\mathfrak{E}$ be two LDFSs, then $d(\mathfrak{D},\mathfrak{E})$ is a distance measure.

Proof

D1) By the definition of LDFS, we get $0 \le a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) \le 1$, $0 \le a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u) \le 1$, $0 \le b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)$ and $0 \le b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u) \le 1$. Then,

$$d(\mathfrak{D},\mathfrak{E}) = \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) \end{bmatrix} \le \frac{1}{2}[1+1] = 1$$

It is obviously that $d(\mathfrak{D},\mathfrak{E}) \ge 0$.

D2) If $\mathfrak{D} = \mathfrak{E}$, then $\mu_{\mathfrak{D}}(u) = \mu_{\mathfrak{E}}(u), \nu_{\mathfrak{D}}(u) = \nu_{\mathfrak{E}}(u), a_{\mathfrak{D}}(u) = a_{\mathfrak{E}}(u), b_{\mathfrak{D}}(u) = b_{\mathfrak{E}}(u)$ for all $u \in \mathfrak{U}$. So,

$$d(\mathfrak{D},\mathfrak{E}) = \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|) \end{bmatrix}$$

$= 0$.

Conversely, if $d(\mathfrak{D},\mathfrak{E}) = 0$, then

$$\max_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) + \max_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) = 0$$

$$|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)| = 0 \ \wedge \ |b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)| = 0$$

Thus, $a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) = a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)$ and $b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) = b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)$. This implies that $\mathfrak{D} = \mathfrak{E}$.

D3)

$$d(\mathfrak{D},\mathfrak{E}) = \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u) - a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|) \end{bmatrix}$$

$= d(\mathfrak{E},\mathfrak{D})$

D4) For any LDFS $\mathfrak{F} = \{(u, \langle\mu_{\mathfrak{F}}(u), \nu_{\mathfrak{F}}(u)\rangle, \langle a_{\mathfrak{F}}(u), b_{\mathfrak{F}}(u)\rangle) : u \in \mathfrak{U}\}$ if $\mathfrak{D} \subseteq \mathfrak{E} \subseteq \mathfrak{F}$, then $\mu_{\mathfrak{D}}(u) \le \mu_{\mathfrak{E}}(u) \le \mu_{\mathfrak{F}}(u), \nu_{\mathfrak{D}}(u) \ge \nu_{\mathfrak{E}}(u) \ge \nu_{\mathfrak{F}}(u), a_{\mathfrak{D}}(u) \le a_{\mathfrak{E}}(u) \le a_{\mathfrak{F}}(u), b_{\mathfrak{D}}(u) \ge b_{\mathfrak{E}}(u) \ge b_{\mathfrak{F}}(u)$ for all $u \in \mathfrak{U}$. So, we have $a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) \le a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u) \le a_{\mathfrak{F}}(u)\mu_{\mathfrak{F}}(u)$ and $b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) \ge b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u) \ge b_{\mathfrak{F}}(u)\nu_{\mathfrak{F}}(u)$.Thus, we have

$$d(\mathfrak{D},\mathfrak{E}) = \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) \end{bmatrix}$$

$$\le \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{F}}(u)\mu_{\mathfrak{F}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{F}}(u)\nu_{\mathfrak{F}}(u)|) \end{bmatrix}$$

$= d(\mathfrak{D},\mathfrak{F})$

It can be shown $d(\mathfrak{D},\mathfrak{F}) \ge d(\mathfrak{E},\mathfrak{F})$ in the same way, which completes the proof.

The following propositions give the various features of the proposed distance measure under the complement of LDFS.

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Proposition 3.3. Let $\mathfrak{D}$ and $\mathfrak{E}$ be two LDFSs, then we get

1. $d(\mathfrak{D}, \mathfrak{E}^c) = d(\mathfrak{D}^c, \mathfrak{E})$;
2. $d(\mathfrak{D}, \mathfrak{E}) = d(\mathfrak{D}^c, \mathfrak{E}^c)$.

Proof.

1. If

$$\mathfrak{D} = \{(u, \langle \mu_{\mathfrak{D}}(u), \nu_{\mathfrak{D}}(u)\rangle, \langle a_{\mathfrak{D}}(u), b_{\mathfrak{D}}(u)\rangle) : u \in \mathfrak{U}\}$$

and

$$\mathfrak{E} = \{(u, \langle \mu_{\mathfrak{E}}(u), \nu_{\mathfrak{E}}(u)\rangle, \langle a_{\mathfrak{E}}(u), b_{\mathfrak{E}}(u)\rangle) : u \in \mathfrak{U}\}$$

be two LDFSs on universal set $\mathfrak{U}$, then

$$\mathfrak{D}^c = \{(u, \langle \nu_{\mathfrak{D}}(u), \mu_{\mathfrak{D}}(u)\rangle, \langle b_{\mathfrak{D}}(u), a_{\mathfrak{D}}(u)\rangle) : u \in \mathfrak{U}\}$$

and

$$\mathfrak{E}^c = \{(u, \langle \nu_{\mathfrak{E}}(u), \mu_{\mathfrak{E}}(u)\rangle, \langle b_{\mathfrak{E}}(u), a_{\mathfrak{E}}(u)\rangle) : u \in \mathfrak{U}\}.$$

So, we get

$$d(\mathfrak{D}, \mathfrak{E}^c) = \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}^c}(u)\mu_{\mathfrak{E}^c}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}^c}(u)\nu_{\mathfrak{E}^c}(u)|) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}^c}(u)\mu_{\mathfrak{D}^c}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}^c}(u)\nu_{\mathfrak{D}^c}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) \end{bmatrix}$$

$$= d(\mathfrak{D}^c, \mathfrak{E})$$

2.

$$d(\mathfrak{D}, \mathfrak{E}) = \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}^c}(u)\nu_{\mathfrak{D}^c}(u) - b_{\mathfrak{E}^c}(u)\nu_{\mathfrak{E}^c}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}^c}(u)\mu_{\mathfrak{D}^c}(u) - a_{\mathfrak{E}^c}(u)\mu_{\mathfrak{E}^c}(u)|) \end{bmatrix}$$

$$= d(\mathfrak{D}^c, \mathfrak{E}^c).$$

Proposition 3.4. Let $\mathfrak{D}$ be an LDFSs, then $d(\mathfrak{D}, \mathfrak{D}^c) = 1$ if and only if $\mathfrak{D}$ is crisp set.

Proof.

If $\mathfrak{D}$ is a crisp set, then $\mu_{\mathfrak{D}}(u) = 1$, $\nu_{\mathfrak{D}}(u) = 0$, $a_{\mathfrak{D}}(u) = 1$ and $b_{\mathfrak{D}}(u) = 0$ or $\mu_{\mathfrak{D}}(u) = 0$, $\nu_{\mathfrak{D}}(u) = 1$, $a_{\mathfrak{D}}(u) = 0$ and $b_{\mathfrak{D}}(u) = 1$. In both cases

$$d(\mathfrak{D}, \mathfrak{D}^c) = \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{D}^c}(u)\mu_{\mathfrak{D}^c}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{D}^c}(u)\nu_{\mathfrak{D}^c}(u)|) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|) \\ + \max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u)|) \end{bmatrix}$$

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$$= \max_{x\in X}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|)$$

$$= 1\,.$$

Otherwise, if $d(\mathfrak{D},\mathfrak{D}^c) = 1$, then we have

$$\frac{1}{2}\left[\begin{matrix}\max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{D}^c}(u)\mu_{\mathfrak{D}^c}(u)|)\\ +\max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{D}^c}(u)\nu_{\mathfrak{D}^c}(u)|)\end{matrix}\right] = 1$$

$$\frac{1}{2}\left[\begin{matrix}\max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|)\\ +\max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u)|)\end{matrix}\right] = 1$$

$$\max_{x\in X}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|) = 1$$

$$|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)| = 1.$$

In this case, either $a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) = 1 \Rightarrow a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) = 1 + b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)$, or $a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) = -1 \Rightarrow b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) = 1 + a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u)$. Since $0 \le a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) + b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) \le 1$, we get $\mathfrak{D}$ is a crisp set.

Proposition 3.5. Let $\mathfrak{D}$ be an LDFSs, then $d(\mathfrak{D},\mathfrak{D}^c) = 0$ if and only if $a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) = b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)$ for all $u \in \mathfrak{U}$.

Proof.

If $a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) = b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)$ for all$u \in \mathfrak{U}$, then

$$d(\mathfrak{D},\mathfrak{D}^c) = \frac{1}{2}\left[\begin{matrix}\max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{D}^c}(u)\mu_{\mathfrak{D}^c}(u)|)\\ +\max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{D}^c}(u)\nu_{\mathfrak{D}^c}(u)|)\end{matrix}\right]$$

$$= \frac{1}{2}\left[\begin{matrix}\max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|)\\ +\max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u)|)\end{matrix}\right]$$

$$= 0.$$

Conversely, if $d(\mathfrak{D},\mathfrak{D}^c) = 0$, then it follows that

$$\frac{1}{2}\left[\begin{matrix}\max\limits_{u\in\mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|)\\ +\max\limits_{u\in\mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u)|)\end{matrix}\right] = 0$$

which implies that

$$|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)| = 0$$

$$|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) = b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u)|.$$

This completes the proof.

Definition 3.6: Let $\mathfrak{D}$ and $\mathfrak{E}$ be two LDFSs. If the mapping $s: LDFS(\mathfrak{U}) \times LDFS(\mathfrak{U}) \to [0,1]$ satisfies the following properties, then s is called a similarity measure between LDFSs $\mathfrak{D}$ and $\mathfrak{E}$.

S1) $0 \le s(\mathfrak{D},\mathfrak{E}) \le 1$,

S2) $s(\mathfrak{D},\mathfrak{E}) = 1$ if and only if $\mathfrak{D} = \mathfrak{E}$,

S3) $s(\mathfrak{D},\mathfrak{E}) = s(\mathfrak{E},\mathfrak{D})$,

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S4) $s(\mathfrak{D}, \mathfrak{D}^c) = 0$ if and only if $\mathfrak{D}$ is crisp set,

S5) If $\mathcal{L} \subseteq \mathfrak{E} \subseteq \mathfrak{F}$, then $s(\mathfrak{D}, \mathfrak{F}) \leq s(\mathfrak{D}, \mathfrak{E})$ and $s(\mathfrak{D}, \mathfrak{F}) \leq s(\mathfrak{E}, \mathfrak{F})$ for all $\mathfrak{D}, \mathfrak{E}, \mathfrak{F} \in LDFS(\mathfrak{U})$.

Theorem 3.7. Let d be a distance measure for LDFSs $\mathfrak{D}$ and $\mathfrak{E}$, then $s(\mathfrak{D}, \mathfrak{E}) = 1 - d(\mathfrak{D}, \mathfrak{E})$ is a similarity measure between $\mathfrak{D}$ and $\mathfrak{E}$.

Proof.

S1) By the Definition of distance $0 \leq d(\mathfrak{D}, \mathfrak{E}) \leq 1$. Thus, we get $0 \leq s(\mathfrak{D}, \mathfrak{E}) \leq 1$.

S2) If $\mathfrak{D} = \mathfrak{E}$, then $d(\mathfrak{D}, \mathfrak{E}) = 0$. Hence $s(\mathfrak{D}, \mathfrak{E}) = 1 - d(\mathfrak{D}, \mathfrak{E}) = 1$. Otherwise, $s(\mathfrak{D}, \mathfrak{E}) = 1 \Rightarrow 1 - d(\mathfrak{D}, \mathfrak{E}) = 1 \Rightarrow d(\mathfrak{D}, \mathfrak{E}) = 0 \Rightarrow \mathfrak{D} = \mathfrak{E}$.

S3) Since $d(\mathfrak{D}, \mathfrak{E}) = d(\mathfrak{E}, \mathfrak{D})$, we get $s(\mathfrak{D}, \mathfrak{E}) = 1 - d(\mathfrak{D}, \mathfrak{E}) = 1 - d(\mathfrak{E}, \mathfrak{D}) = s(\mathfrak{E}, \mathfrak{D})$.

S4) If $\mathfrak{D}$ is crisp set, then $s(\mathfrak{D}, \mathfrak{D}^c) = 1 - d(\mathfrak{D}, \mathfrak{D}^c) = 0$, by Proposition 3.4. Conversely, $s(\mathfrak{D}, \mathfrak{D}^c) = 0 \Rightarrow 1 - d(\mathfrak{D}, \mathfrak{D}^c) = 0 \Rightarrow d(\mathfrak{D}, \mathfrak{D}^c) = 1$, so $\mathfrak{D}$ is crisp set.

S5) If $\mathfrak{D} \subseteq \mathfrak{E} \subseteq \mathfrak{F}$, then $d(\mathfrak{D}, \mathfrak{F}) \geq d(\mathfrak{D}, \mathfrak{E})$ and $d(\mathfrak{D}, \mathfrak{F}) \geq d(\mathfrak{E}, \mathfrak{F})$ for all $\mathfrak{D}, \mathfrak{E}, \mathfrak{F} \in LDFS(\mathfrak{U})$, by the definition of distance measure. Hence $s(\mathfrak{D}, \mathfrak{F}) = 1 - d(\mathfrak{D}, \mathfrak{F}) \leq 1 - d(\mathfrak{D}, \mathfrak{E}) = s(\mathfrak{D}, \mathfrak{E})$ and $s(\mathfrak{D}, \mathfrak{F}) = 1 - d(\mathfrak{D}, \mathfrak{F}) \leq 1 - d(\mathfrak{E}, \mathfrak{F}) = s(\mathfrak{E}, \mathfrak{F})$ for all $\mathfrak{D}, \mathfrak{E}, \mathfrak{F} \in LDFS(\mathfrak{U})$.

For $\mathfrak{D}, \mathfrak{E} \in LDFS(\mathfrak{U})$, we have

$$s_d(\mathfrak{D}, \mathfrak{E}) = 1 - d(\mathfrak{D}, \mathfrak{E}) = 1 - \frac{1}{2}\left[\begin{array}{l}\max\limits_{u \in \mathfrak{U}}(|a_{\mathfrak{D}}(u)\mu_{\mathfrak{D}}(u) - a_{\mathfrak{E}}(u)\mu_{\mathfrak{E}}(u)|) \\ + \max\limits_{u \in \mathfrak{U}}(|b_{\mathfrak{D}}(u)\nu_{\mathfrak{D}}(u) - b_{\mathfrak{E}}(u)\nu_{\mathfrak{E}}(u)|)\end{array}\right].$$

CONCLUSION

Linear Diophantine fuzzy theory was developed with the aim of overcoming the uncertainty due to the use of reference parameters in [5]. It also appears that the LDFS concept with reference parameters has a wider variety of fuzzy data then the theories of IFS, PyFS and q-ROFS. In this work, we proposed new distance measure for linear Diophantine fuzzy sets and gave its properties. Moreover, we shown the relationship between the distance and similarity measures for LDFSs. Also, we introduced new similarity measure for LDFSs based on proposed distance measure.

Conflict of Interest

The authors declare that they have no conflict of interest.

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