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A new family of Gauss (k,t)-Horadam numbers

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We present a new family of Gauss (k,t)-Horadam numbers and obtain Binet formula of this family. We give the relationship between this family and the known Gauss t-Horadam number. Then we prove the Cassini and Catalan identities for this family. Furthermore, we investigate the sums, the recurrence relations and generating functions of this family.

Keywords: Cassini identity; Fibonacci number; Gauss number; generating function; Horadam number.

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1. Introduction

Many authors are interested in Fibonacci, Lucas, Pell, Jacobsthal, Pell–Lucas, Jacobsthal–Lucas, generalized k-Fibonacci and k-Lucas numbers, etc. For interesting application of these numbers in science and nature, one can see [1–3, 5, 13, 15]. Mikkawy presented a new family of k-Fibonacci numbers and gave several properties of this family [7]. Öcal *et al.* introduced some determinantal and the permanental representations of k-generalized Fibonacci and Lucas numbers [8]. Özkan *et al.* carried on Mikkawy work to Lucas numbers [9].

There are many works related to Horadam numbers and their applications [6, 18, 19]. Tan and Leung presented some matrix representations associated with the Horadam quaternions [16].

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Some studies have also been done on the Gauss-Fibonacci and Lucas numbers [10–12, 14, 17]. One of the latest works is [12] where they define a new family of Gauss k-Lucas numbers and the generalized polynomials for these numbers.

Gökbaş and Köse define the complex k-Horadam and Gaussian k-Horadam sequences and find the distance between complex k-Horadam and Gaussian k-Horadam numbers [4].

Now, we present the generalized t-Horadam $H_{t,n}$ and Gauss t-Horadam numbers $\mathrm{GH}_{t,n}$.

Let t be any positive real number and $f(t), g(t)$ be scaler-value polynomials. For $n \in \mathbb{N}$ and $f^2(t) + 4g(t) > 0$, $H_{t,n}$ is defined by

$$H_{t,n+2} = f(t)H_{t,n+1} + g(t)H_{t,n}, \tag{1.1}$$

where $H_{t,0} = a$, $H_{t,1} = b$ [19].

- The Binet formula of $H_{t,n}$:

$$H_{t,n} = \frac{c\alpha^n - d\beta^n}{\alpha - \beta}, \tag{1.2}$$

- The Cassini identity of $H_{t,n}$:

$$H_{t,n-1}H_{t,n+1} - (H_{t,n})^2 = (-g(t))^{n-1}(a^2g(t) + abf(t) - b^2), \tag{1.3}$$

- The property of $H_{t,n}$:

$$\begin{aligned} &H_{t,n}H_{t,n-r+s} - H_{t,n-r}H_{t,n+s} \\ &\quad = \frac{(-g(t))^{n-r}(bH_{t,r} - aH_{t,r+1})(bH_{t,m-n+r} - aH_{t,m-n+r+1})}{b^2 - a^2g(t) - abf(t)}, \end{aligned} \tag{1.4}$$

where $c = b - a\beta$, $d = b - a\alpha$, $\alpha = \frac{f(t)+\sqrt{f^2(t)+4g(t)}}{2}$ and $\beta = \frac{f(t)-\sqrt{f^2(t)+4g(t)}}{2}$ [19].

For $n \in \mathbb{N}$ and $f^2(t) + 4g(t) > 0$, $\mathrm{GH}_{t,n}$ is defined by

$$\mathrm{GH}_{t,n} = H_{t,n} + iH_{t,n-1}, \tag{1.5}$$

where $\mathrm{GH}_{t,1} = b + ai$, $\mathrm{GH}_{t,2} = bf(t) + ag(t) + ib$. So we have

$$\mathrm{GH}_{t,n+2} = f(t)\mathrm{GH}_{t,n+1} + g(t)\mathrm{GH}_{t,n} \tag{1.6}$$

[4].

In this work, we define a new family of Gauss (k, t)-Horadam numbers and obtain the Binet formula of this family. Then, we get the Cassini and Catalan identities for the Gauss t-Horadam numbers. We give the relationships between this family and the Gauss t-Horadam numbers. Also, we prove the Cassini identity for this family. Finally, we find the sums, the recurrence relations and generating functions for this family.

2. Main Results

Lemma 1. *For $n \geq 1$, the Cassini identity for $\mathrm{GH}_{t,n}$ has the form*

$$\mathrm{GH}_{t,n-1}\mathrm{GH}_{t,n+1} - (\mathrm{GH}_{t,n})^2 = (-g(t))^{n-2}(b^2 - a^2 g(t) - abf(t))(1 + g(t) - if(t)).$$

Proof. The proof can be seen using Eqs. (1.3)–(1.5). □

Definition 1. Let $m(\neq 0), k(\neq 0), r$ be any natural numbers such that $n = mk + r$ and $0 \leq r < k$. The family of Gauss (k,t)-Horadam numbers is defined by

$$\mathrm{GH}_{t,n}^{(k)} = \sum_{i=1}^{r} \binom{r}{i} (f(t)\mathrm{GH}_{t,m})^{k-i} (g(t)\mathrm{GH}_{t,m-1})^{i}. \tag{2.1}$$

The Binet formula for $\mathrm{GH}_{t,n}^{(k)}$ is

$$\mathrm{GH}_{t,n}^{(k)} = \left[\frac{c\alpha^m - d\beta^m}{\alpha - \beta} + i\frac{c\alpha^{m-1} - d\beta^{m-1}}{\alpha - \beta}\right]^{k-r} \times \left[\frac{c\alpha^{m+1} - d\beta^{m+1}}{\alpha - \beta} + i\frac{c\alpha^m - d\beta^m}{\alpha - \beta}\right]^{r}, \tag{2.2}$$

where c, d, α and β are as follows in Eq. (1.2).

From Eqs. (1.2), (1.5) and (2.2), we can give this family via the Gauss t-Horadam numbers. So we obtain

$$\mathrm{GH}_{t,n}^{(k)} = (\mathrm{GH}_{t,m})^{k-r}(\mathrm{GH}_{t,m+1})^{r}, \quad n = mk + r. \tag{2.3}$$

Additionally, we can express the family of Gauss (k,t)-Horadam numbers by matrix methods. Indeed, we have

$$\mathrm{GH}_{t,n}^{k-1} gh_n = \begin{bmatrix} \mathrm{GH}_{t,kn+1}^{(k)} & \mathrm{GH}_{t,kn}^{(k)} \\ \mathrm{GH}_{t,kn}^{(k)} & \mathrm{GH}_{t,kn-1}^{(k)} \end{bmatrix},$$

where $gh_n = \begin{bmatrix} \mathrm{GH}_{t,n+1} & \mathrm{GH}_{t,n} \\ \mathrm{GH}_{t,n} & \mathrm{GH}_{t,n-1} \end{bmatrix}$, $n \geq 1$.

In all the conclusions of Sec. 2, to reveal the importance of this issue, we can state specific and close relationships as follows:

- If $f(t) = g(t) = 1$, $a = 0$, $b = 1$, then the family of Gauss Fibonacci numbers $\mathrm{GF}_n^{(k)}$ is obtained,
- If $f(t) = 2$, $g(t) = 1$, $a = 0$, $b = 1$, then the family of Gauss Pell numbers $\mathrm{GP}_n^{(k)}$ is obtained,
- If $f(t) = 1$, $g(t) = 2$, $a = 0$, $b = 1$, then the family of Gauss Jacobsthal numbers $\mathrm{GJ}_n^{(k)}$ is obtained,
- If $f(t) = g(t) = 1$, $a = 2$, $b = 1$, then the family of Gauss Lucas numbers $\mathrm{GL}_n^{(k)}$ is obtained,

- If $f(t) = 2$, $g(t) = 1$, $a = 2$, $b = 2$, then the family of Gauss Pell–Lucas numbers $\mathrm{GQ}_n^{(k)}$ is obtained,
- If $f(t) = 1$, $g(t) = 2$, $a = 2$, $b = 1$, then the family of Gauss Jacobsthal–Lucas numbers $Gj_n^{(k)}$ is obtained,
- If $f(t) = p$, $g(t) = q$, $a = 0$, $b = 1$, then the family of Gauss k-Fibonacci numbers $\mathrm{GU}_n^{(k)}$ is obtained,
- If $f(t) = p$, $g(t) = q$, $a = 2$, $b = p$, then the family of Gauss k-Lucas numbers $\mathrm{GV}_n^{(k)}$ is obtained,
- If $f(t) = s$, $g(t) = t$, $a = 0$, $b = 1$, then the family of (s,t)-Gauss Fibonacci numbers $\mathrm{GF}_{s,t,n}^{(k)}$ is obtained,
- If $f(t) = s$, $g(t) = t$, $a = 2$, $b = 1$, then the family of (s,t)-Gauss Lucas numbers $\mathrm{GL}_{s,t,n}^{(k)}$ is obtained.

Theorem 1. *For $k = 2$, we have the following recurrence relation and the generating function for the family of Gauss (k,t)-Horadam numbers*:

(i) $\mathrm{GH}_{t,n}^{(2)} = f(t)\mathrm{GH}_{t,n-1}^{(2)} + f(t)g(t)\mathrm{GH}_{t,n-3}^{(2)} + g^2(t)\mathrm{GH}_{t,n-4}^{(2)}$,

(ii) $G^{(2)}(t,x) = \frac{x^2(b+ai)^2(b+ai+bf(t)x+ag(t)x+bix-aif(t)x)}{1-f(t)x-f(t)g(t)x^3-g^2(t)x^4}$.

Proof. (i) There are two cases of subscript n. If n is even integer, by using Eqs. (1.6) and (2.3), we have

$$\begin{aligned}\mathrm{GH}_{t,2m+4}^{(2)} &= (\mathrm{GH}_{t,m+2})^2\\ &= \mathrm{GH}_{t,m+2}(f(t)\mathrm{GH}_{t,m+1} + g(t)\mathrm{GH}_{t,m})\\ &= f(t)\mathrm{GH}_{t,m+1}\mathrm{GH}_{t,m+2} + g(t)\mathrm{GH}_{t,m}(f(t)\mathrm{GH}_{t,m+1} + g(t)\mathrm{GH}_{t,m})\\ &= f(t)\mathrm{GH}_{t,2m+3}^{(2)} + f(t)g(t)\mathrm{GH}_{t,2m+1}^{(2)} + g^2(t)\mathrm{GH}_{t,2m}^{(2)}.\end{aligned}$$

If n is odd integer, then we have

$$\begin{aligned}\mathrm{GH}_{t,2m+3}^{(2)} &= \mathrm{GH}_{t,m+1}\mathrm{GH}_{t,m+2}\\ &= \mathrm{GH}_{t,m+1}(f(t)\mathrm{GH}_{t,m+1} + g(t)\mathrm{GH}_{t,m})\\ &= f(t)\mathrm{GH}_{t,m+1}^2 + g(t)\mathrm{GH}_{t,m}(f(t)\mathrm{GH}_{t,m} + g(t)\mathrm{GH}_{t,m-1})\\ &= f(t)\mathrm{GH}_{t,2m+2}^{(2)} + f(t)g(t)\mathrm{GH}_{t,2m}^{(2)} + g^2(t)\mathrm{GH}_{t,2m-1}^{(2)}.\end{aligned}$$

(ii) Let $G^{(2)}(t,x)$ be generating function for the family of Gauss $(2,t)$-Horadam numbers, where

$$G^{(2)}(x) = \sum_{n=0}^{\infty} \mathrm{GH}_{t,n}^{(2)} x^n. \tag{2.4}$$

If $G^{(2)}(x)$ given in (2.4) is multiplied with $f(t)x$, $f(t)g(t)x^3$ and $g^2(t)x^4$, respectively, then we get

$$\left.\begin{aligned} f(t)xG^{(2)}(x) &= f(t)\sum_{n=1}^{\infty}\mathrm{GH}^{(2)}_{t,n-1}x^n,\\ f(t)g(t)x^3G^{(2)}(x) &= f(t)g(t)\sum_{n=3}^{\infty}\mathrm{GH}^{(2)}_{t,n-3}x^n,\\ g^2(t)x^4G^{(2)}(x) &= g^2(t)\sum_{n=4}^{\infty}\mathrm{GH}^{(2)}_{t,n-4}x^n. \end{aligned}\right\} \tag{2.5}$$

Consequently, by subtracting (2.5) from (2.4), we obtain the equation

$$G^{(2)}(x)=\frac{x^2(b+ai)^2(b+ai+bf(t)x+ag(t)x+bix-aif(t)x)}{1-f(t)x-f(t)g(t)x^3-g^2(t)x^4},$$

as required. □

Theorem 2. *We have the following recurrence relation and the generating function for the family of Gauss (k,t)-Horadam numbers*:

(i) $\mathrm{GH}^{(k)}_{t,mk+1}=f(t)\mathrm{GH}^{(k)}_{t,mk}+g(t)\mathrm{GH}^{(k)}_{t,mk-1}$,

(ii) $\sum_{mk+1=0}^{\infty}\mathrm{GH}^{(k)}_{t,mk+1}x^{mk+1}=\frac{\mathrm{GH}^{(k)}_{t,1}x}{1-f(t)x-g(t)x^2}$.

Proof. The proof is done similarly to that of Theorem 1. □

Theorem 3. *For $m,k(\neq 0)$, we have*

$$(\mathrm{GH}_{t,m+1})^k-(\mathrm{GH}_{t,m})^k=\mathrm{GH}^{(k)}_{t,(m+1)k}-\mathrm{GH}^{(k)}_{t,mk}.$$

Proof. From Eq. (2.3), we acquire

$$\begin{aligned}\mathrm{GH}^{(k)}_{t,(m+1)k}-\mathrm{GH}^{(k)}_{t,mk}&=(\mathrm{GH}_{t,m})^{k-k}(\mathrm{GH}_{t,m+1})^k-(\mathrm{GH}_{t,m})^{k-0}(\mathrm{GH}_{t,m+1})^0\\&=(\mathrm{GH}_{t,m+1})^k-(\mathrm{GH}_{t,m})^k.\end{aligned}$$

□

Let us suppose that $\mathrm{GH}^{(k)}_{t,-n}=0$ for $k=1,2,\ldots$.

Theorem 4. *For $n,s\geq 0$, $n+s\geq 1$, we have*

$$\begin{aligned}\mathrm{GH}_{t,n+s}\mathrm{GH}_{t,n+s-2}-\mathrm{GH}^{(2)}_{t,2(n+s-1)}&=(-g(t))^{n+s-3}(b^2-a^2g(t)-abf(t))\\&\quad\times(1+g(t)-if(t)).\end{aligned}$$

Proof. From Eq. (2.3) and Lemma 1, we get

$$\begin{aligned}\mathrm{GH}_{t,n+s}\mathrm{GH}_{t,n+s-2}-\mathrm{GH}^{(2)}_{t,2(n+s-1)}&=\mathrm{GH}_{t,n+s}\mathrm{GH}_{t,n+s-2}-\mathrm{GH}^{2}_{t,n+s-1},\\&=(-g(t))^{n+s-3}(b^2-a^2g(t)-abf(t))\\&\quad\times(1+g(t)-if(t)).\end{aligned}$$

□

Theorem 5. *Let s, n, k be integer numbers such that $1 \leq s < k, n, k \geq 2$, the Cassini Identity for $\mathrm{GH}_{t,n}^{(k)}$ is as follows*:

$$\mathrm{GH}_{t,kn+s}^{(k)}\mathrm{GH}_{t,kn+s-2}^{(k)} - (\mathrm{GH}_{t,kn+s-1}^{(k)})^2 = \begin{cases} \mathrm{GH}_{t,n}^{2k-2}(-g(t))^{n-2}(b^2 - a^2g(t) - abf(t))(1 + g(t) - if(t)), & s = 1, \\ 0, & s \neq 1. \end{cases}$$

Proof. For $s = 1$, from (2.3) and Lemma 1, we obtain

$$\begin{aligned} &\mathrm{GH}_{t,kn+1}^{(k)}\mathrm{GH}_{t,kn-1}^{(k)} - (\mathrm{GH}_{t,kn}^{(k)})^2 \\ &\quad = (\mathrm{GH}_{t,n}^{k-1}\mathrm{GH}_{t,n+1})(\mathrm{GH}_{t,n-1}\mathrm{GH}_{t,n}^{k-1}) - (\mathrm{GH}_{t,n}^{k})^2 \\ &\quad = \mathrm{GH}_{t,n}^{2k-2}(\mathrm{GH}_{t,n+1}\mathrm{GH}_{t,n-1} - (\mathrm{GH}_{t,n})^2) \\ &\quad = \mathrm{GH}_{t,n}^{2k-2}(-g(t))^{n-2}(b^2 - a^2g(t) - abf(t))(1 + g(t)) - if(t)). \end{aligned}$$

For $s \neq 1$, we have

$$\begin{aligned} &\mathrm{GH}_{t,kn+s}^{(k)}\mathrm{GH}_{t,kn+s-2}^{(k)} - (\mathrm{GH}_{t,kn+s-1}^{(k)})^2 \\ &\quad = (\mathrm{GH}_{t,n}^{k-s}\mathrm{GH}_{t,n+1}^{s})(\mathrm{GH}_{t,n}^{k-s+2}\mathrm{GH}_{t,n+1}^{s-2}) - (\mathrm{GH}_{t,n}^{k-s+1}\mathrm{GH}_{t,n+1}^{s-1})^2 \\ &\quad = \mathrm{GH}_{t,n}^{2k-2s+2}(\mathrm{GH}_{t,n+1}^{2s-2} - \mathrm{GH}_{t,n+1}^{2s-2}) = 0. \end{aligned}$$

□

Theorem 6. *Let s, n, k be integer numbers such that $0 \leq 3s < k, n, k \geq 2$, the Catalan Identity for $\mathrm{GH}_{t,n}^{(k)}$ is as follows*:

$$\mathrm{GH}_{t,kn+3s}^{(k)}\mathrm{GH}_{t,kn+s}^{(k)} - (\mathrm{GH}_{t,kn+2s}^{(k)})^2 = 0.$$

Proof. By using Eq. (2.3), we obtain

$$\begin{aligned} &\mathrm{GH}_{t,kn+3s}^{(k)}\mathrm{GH}_{t,kn+s}^{(k)} - (\mathrm{GH}_{t,kn+2s}^{(k)})^2 \\ &\quad = (\mathrm{GH}_{t,n}^{k-3s}\mathrm{GH}_{t,n+1}^{3s})(\mathrm{GH}_{t,n}^{k-s}\mathrm{GH}_{t,n+1}^{s}) - \mathrm{GH}_{t,n}^{2k-4s}\mathrm{GH}_{t,n+1}^{4s} = 0. \end{aligned}$$

□

Proposition 1. *For any n, we obtain the following properties for $\mathrm{GH}_{t,n}^{(2)}$:*

(i) $\mathrm{GH}_{t,2n+4}^{(2)} - f^2(t)\mathrm{GH}_{t,2n+2}^{(2)} = 2f(t)g(t)\mathrm{GH}_{t,2n+1}^{(2)} + g^2(t)\mathrm{GH}_{t,2n}^{(2)}$,

(ii) $f(t)\mathrm{GH}_{t,2n+3}^{(2)} + g(t)\mathrm{GH}_{t,2n+2}^{(2)} = H_{t,2n}^{(2)} + (-g(t))^{n-1}(a^2g(t) + abf(t) - b^2)$.

Proof. (i) By using Eqs. (2.3) and (1.6), we obtain

$$\begin{aligned} &\mathrm{GH}_{t,2n+4}^{(2)} - f^2(t)\mathrm{GH}_{t,2n+2}^{(2)} \\ &\quad = \mathrm{GH}_{t,n+2}^{2} - (f(t)\mathrm{GH}_{t,n+1})^2 \end{aligned}$$

$$\begin{aligned}
&= (\mathrm{GH}_{t,n+2} - f(t)\mathrm{GH}_{t,n+1})(\mathrm{GH}_{t,n+2} + f(t)\mathrm{GH}_{t,n+1}) \\
&= g(t)\mathrm{GH}_{t,n}(2f(t)\mathrm{GH}_{t,n+1} + g(t)\mathrm{GH}_{t,n}) \\
&= 2f(t)g(t)\mathrm{GH}_{t,n}\mathrm{GH}_{t,n+1} + g^2(t)\mathrm{GH}_{t,n}^2 \\
&= 2f(t)g(t)\mathrm{GH}_{t,2n+1}^{(2)} + g^2(t)\mathrm{GH}_{t,2n}^{(2)}.
\end{aligned}$$

(ii) By using Eqs. (2.3) and (1.6), we obtain

$$\begin{aligned}
f(t)\mathrm{GH}_{t,2n+3}^{(2)} + g(t)\mathrm{GH}_{t,2n+2}^{(2)} &= f(t)\mathrm{GH}_{t,n+1}\mathrm{GH}_{t,n+2} + g(t)(\mathrm{GH}_{t,n+1})^2 \\
&= \mathrm{GH}_{t,n+1}(f(t)\mathrm{GH}_{t,n+2} + g(t)\mathrm{GH}_{t,n+1}) \\
&= \mathrm{GH}_{t,n+1}\mathrm{GH}_{t,n+3}.
\end{aligned}$$

Then, from Lemma 1 and Eq. (2.3), we get

$$\begin{aligned}
&f(t)\mathrm{GH}_{t,2n+3}^{(2)} + g(t)\mathrm{GH}_{t,2n+2}^{(2)} \\
&\quad = (\mathrm{GH}_{t,n+2})^2 + (-g(t))^n(b^2 - a^2g(t) - abf(t))(1 + g(t) - if(t)) \\
&\quad = \mathrm{GH}_{t,2n+4}^{(2)} + (-g(t))^n(b^2 - a^2g(t) - abf(t))(1 + g(t) - if(t)).
\end{aligned}$$

□

Theorem 7. *Sums for the family of Gauss (k,t)-Horadam numbers are*

(i) $\sum_{i=0}^{k-1}(f(t))^{-i}\mathrm{GH}_{t,mk+i}^{(k)} = \frac{\mathrm{GH}_{t,m}}{\mathrm{GH}_{t,m-1}}\left(\frac{\mathrm{GH}_{t,(m+1)k}^{(k)} - (f(t))^k\mathrm{GH}_{t,mk}^{(k)}}{f^{k-1}(t)g(t)}\right)$,

(ii) $g^{k-1}(t)\sum_{i=0}^{k-1}\left(\frac{f(t)}{g(t)}\right)^i\binom{k-1}{i}\mathrm{GH}_{t,mk+i}^{(k)} = \mathrm{GH}_{t,m}\mathrm{GH}_{t,(m+2)(k-1)}^{(k-1)}$,

(iii) $\sum_{i=0}^{k-1}(-1)^i(f(t))^{k-i-1}\binom{k-1}{i}\mathrm{GH}_{t,mk+i}^{(k)} = (g(t))^{k-1}\mathrm{GH}_{t,m}\mathrm{GH}_{t,(m-1)(k-1)}^{(k-1)}$.

Proof. (i) By considering Eqs. (2.3) and (1.6), we can write

$$\begin{aligned}
\sum_{i=0}^{k-1}(f(t))^{-i}\mathrm{GH}_{t,mk+i}^{(k)} &= \sum_{i=0}^{k-1}\left(\frac{\mathrm{GH}_{t,m+1}}{f(t)\mathrm{GH}_{t,m}}\right)^i\mathrm{GH}_{t,m}^k \\
&= \mathrm{GH}_{t,m}^k\left(\frac{\left(\frac{\mathrm{GH}_{t,m+1}}{f(t)\mathrm{GH}_{t,m}}\right)^k - 1}{\frac{\mathrm{GH}_{t,m+1}}{f(t)\mathrm{GH}_{t,m}} - 1}\right) \\
&= \frac{\mathrm{GH}_{t,m}}{f^{k-1}(t)g(t)\mathrm{GH}_{t,m-1}}(\mathrm{GH}_{t,m+1}^k - f^k(t)\mathrm{GH}_{t,m}^k) \\
&= \frac{\mathrm{GH}_{t,m}}{\mathrm{GH}_{t,m-1}}\left(\frac{\mathrm{GH}_{t,(m+1)k}^{(k)} - f^k(t)\mathrm{GH}_{t,mk}^{(k)}}{f^{k-1}(t)g(t)}\right).
\end{aligned}$$

The proofs (ii) and (iii) are similar to that of (i). □

3. Conclusion

In this paper, we defined a new family of Gauss (k,t)-Horadam numbers. We gave some relations among this family and the known Gauss t-Horadam numbers. Then we proved Cassini and Catalan identities for the Gauss t-Horadam numbers and this family. Furthermore, we found the recurrence relations, generating functions and summations for this family.

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